Appendix : Navigating Speculative Bubbles: A Higher-Moment Approach to Optimal Asset Allocation

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1. Monte-Carlo Simulations

This section provides additional results complementing the simulation analysis presented in the main text. We focus on (i) the sensitivity of optimal strategy pairs (ω^*, h^*) to parameter estimation (Subsection 1.1), and (ii) the economic value of the proposed portfolio allocation strategies under alternative utility specifications (Subsection 1.2).

Overall, the findings reported here are fully consistent with those discussed in the main paper and further illustrate the robustness of the state-dependent allocation mechanism.

1.1. Optimal strategy pairs sensitivity to parameter estimation

This subsection provides additional evidence on the impact of parameter estimation on optimal portfolio strategies. In particular, we assess how estimation uncertainty affects the mapping between the state variable and the optimal allocation pairs (ω^*, h^*).

Consistent with the results reported in the main paper, the estimated optimal strategies remain tightly clustered around their theoretical counterparts, highlighting the robustness of the plug-in approach even in regions of the state space characterized by extreme values.

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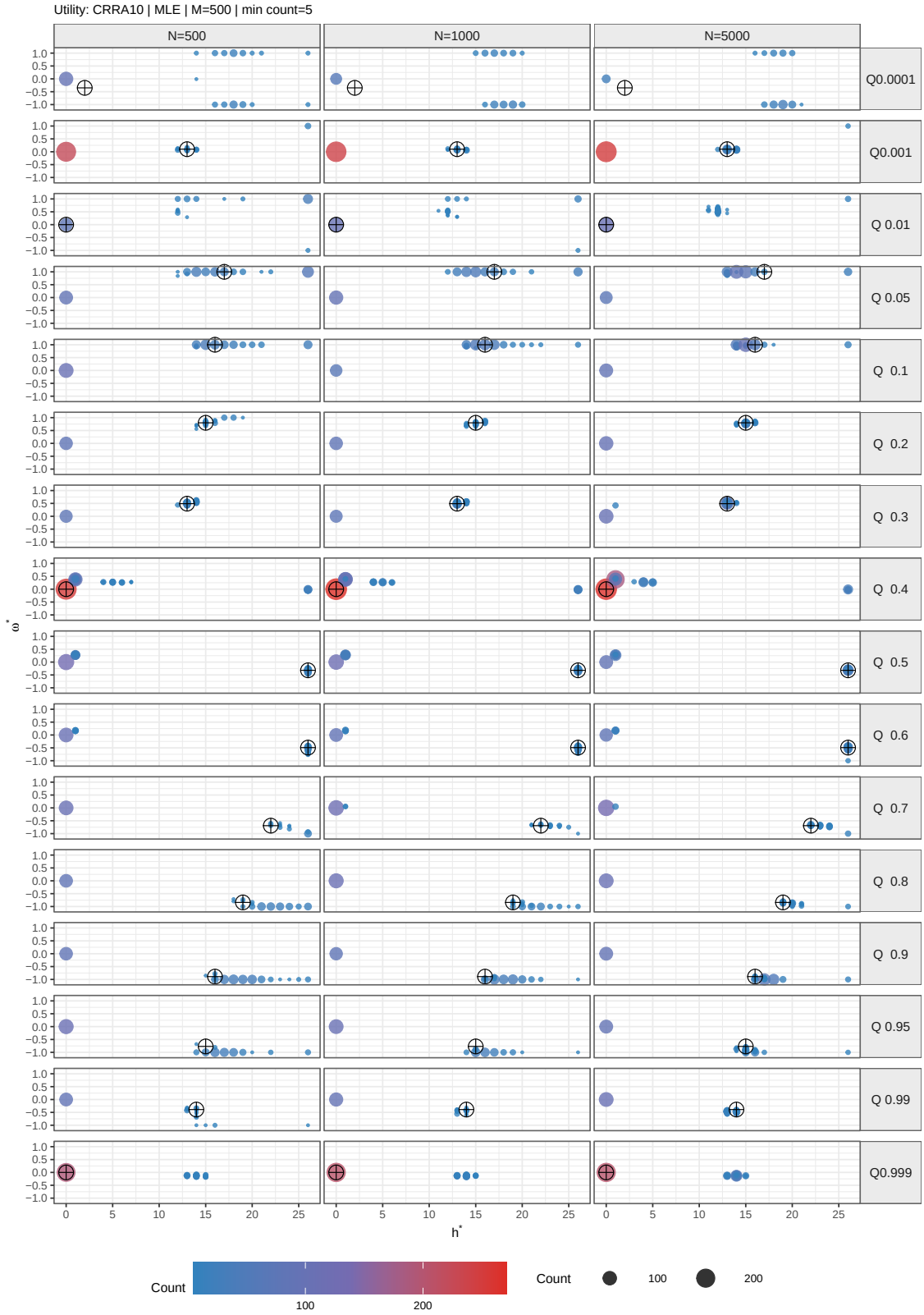
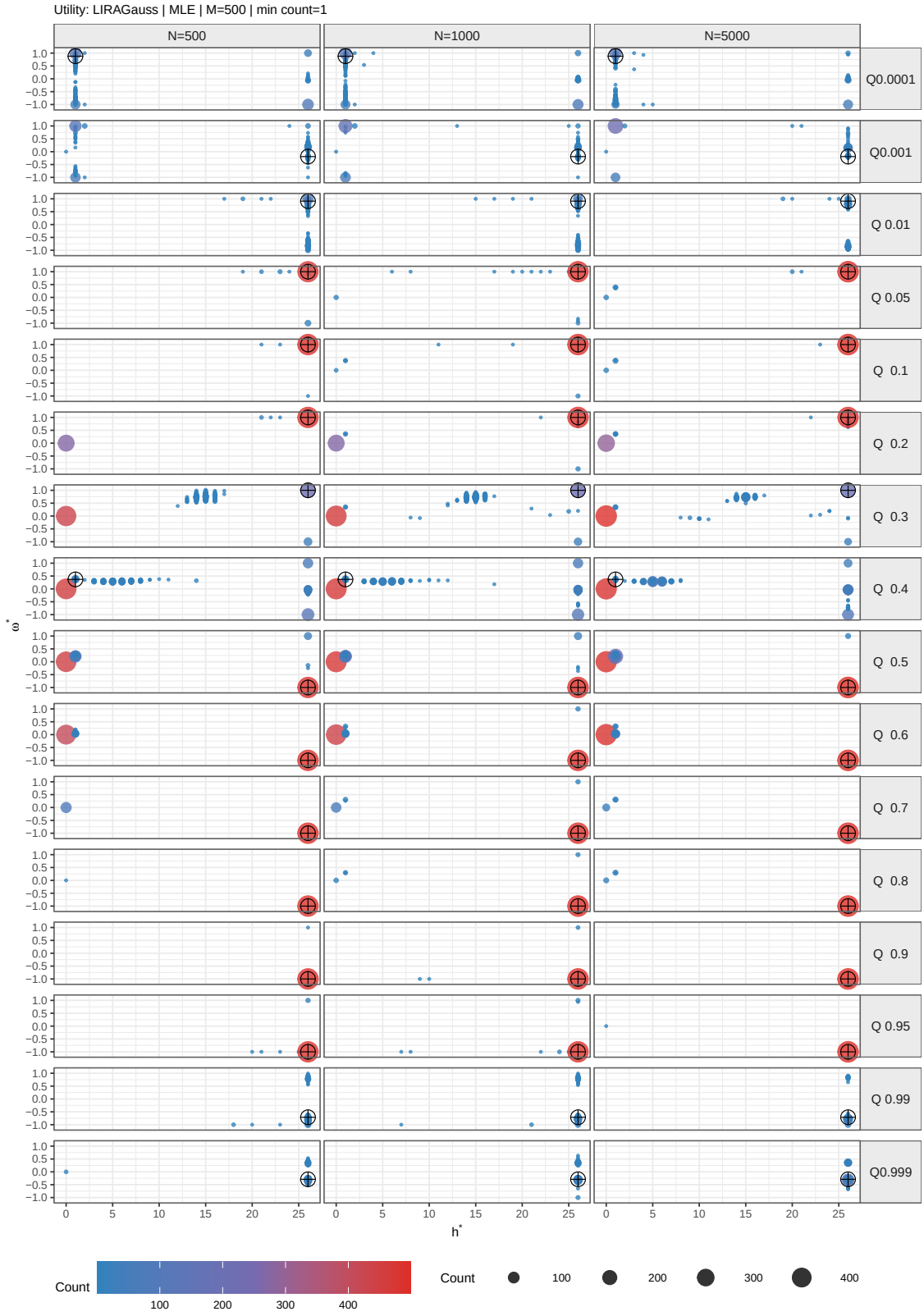
Figure 1: Optimal portfolio strategies (CRRA $\gamma = 10$ utility)



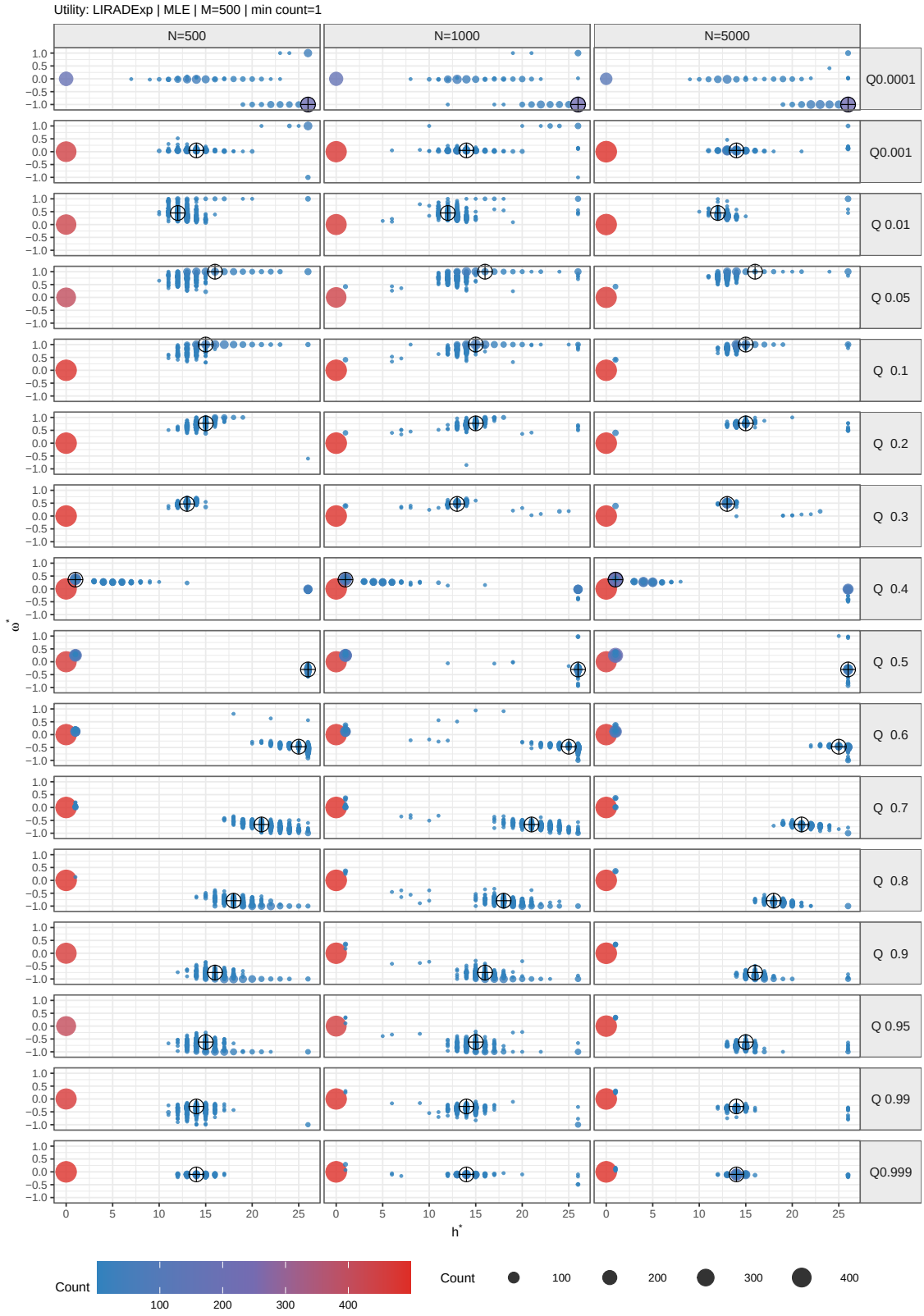
Notes: Mass repartition of the optimal portfolio strategies when the speculative asset's parameters are estimated by MLE across 500 simulated trajectories of length $N = 500, 1000, 5000$ trading days and for several starting values defined by the quantiles of the true marginal distribution of X_t. The DGP for the speculative asset price is a MAR(1,1) process $(1 - 0.9F)(1 - 0.1B)X_t = \varepsilon_t$ with $\varepsilon_t \overset{i.i.d.}{\sim} S(1.7, 0.3, 1.5, 15)$. The results are displayed in the share (vertical axis) - horizon (horizontal axis) space. The larger and redder the dots, the bigger the proportion of selected portfolios falling in that area across the simulations. A black target symbol indicates a true optimal portfolio, i.e. obtained for the true values of the parameters.

Figure 2: Optimal portfolio strategies (LIRA Gaussian utility)



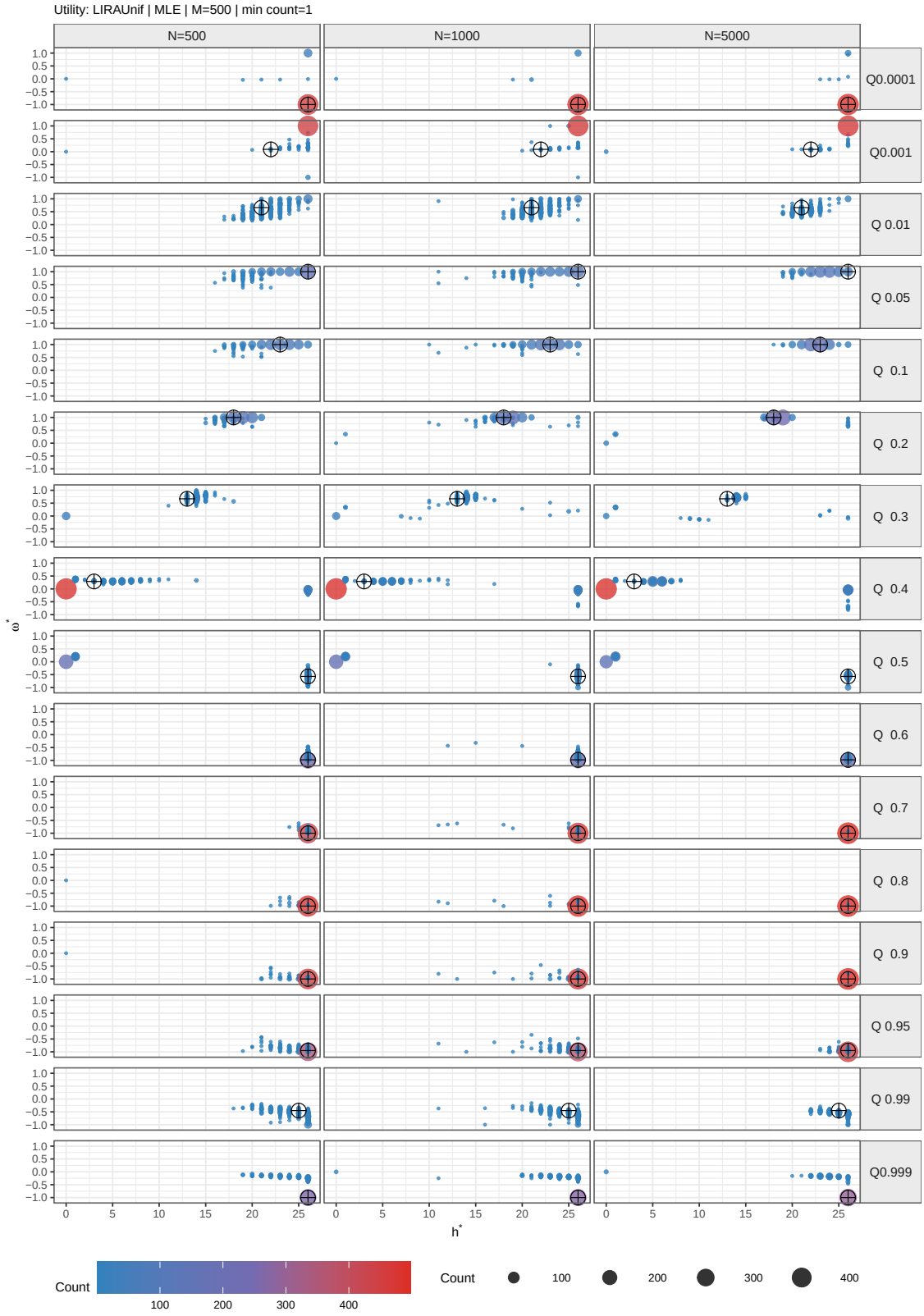
Notes: See note to figure 1.

Figure 3: Optimal portfolio strategies (LIRA Double Exponential utility)



Notes: See note to figure 1.

Figure 4: Optimal portfolio strategies (LIRA Uniform utility)



Notes: See note to figure 1.

1.2. BP strategies based on the first four moments (BP)

The BP strategies reported in Tables 1 to 4 are based on the first four conditional moments of the return distribution, and are derived from the MAR(1,1) dynamics of the speculative asset.

We report results for different preference specifications, including CRRA utility with $\gamma = 10$ and several LIRA specifications (Gaussian, double exponential, and uniform). These alternative specifications allow us to assess the robustness of the portfolio allocation outcomes with respect to risk preferences.

Across all cases, the results confirm that incorporating higher-order moments leads to improved portfolio performance relative to standard benchmark strategies, particularly in environments characterized by asymmetric and heavy-tailed return distributions.

Table 1: Relative performance of portfolio strategies — CRRA ($\gamma = 10$)

			Strategy				
Sample	Metric		BP_{med}	BP_{inf}	BP_{sup}	EW	MV
Full sample	Wealth	μ	$1.024^{***/***}$	$1.023^{***/***}$	$1.025^{***/***}$	1.005	1.010
		σ	0.006	0.007	0.006	0.003	0.003
	OC vs EW	μ	0.021	0.020	0.022		
		σ	0.008	0.008	0.008		
	OC vs MV	μ	0.017	0.016	0.018		
		σ	0.007	0.007	0.006		
Bubble periods	Wealth	μ	$1.063^{***/***}$	$1.062^{***/***}$	$1.064^{***/***}$	0.962	1.005
		σ	0.029	0.030	0.029	0.016	0.011
	OC vs EW	μ	0.097	0.096	0.099		
		σ	0.037	0.037	0.036		
	OC vs MV	μ	0.054	0.053	0.056		
		σ	0.030	0.030	0.029		

Notes: MAR(1,1)-based strategies are compared with equally weighted (EW) and mean-variance (MV) benchmarks. Results are reported for the full sample and for bubble periods ($X_t \geq Q_{95\%}$). Terminal wealth and opportunity cost (OC) are summarized by their mean (μ) and standard deviation (σ). Asterisks $^{*/**/***}$ indicate rejection of one-sided Wilcoxon tests (BP $>$ benchmark) at the 90%, 95%, and 99% levels versus EW/MV, respectively.

Table 2: Relative performance of portfolio strategies — LIRA Gaussian utility

Sample	Metric		Strategy				
			BP_{med}	BP_{inf}	BP_{sup}	EW	MV
Full sample	Wealth	μ	1.023***/***	1.014***/***	1.035***/***	1.005	1.010
		σ	0.008	0.009	0.011	0.003	0.003
	OC vs EW	μ	0.025	0.017	0.035		
		σ	0.011	0.011	0.012		
	OC vs MV	μ	0.021	0.013	0.031		
		σ	0.011	0.011	0.012		
Bubble periods	Wealth	μ	1.089***/***	1.088***/***	1.091***/***	0.962	1.005
		σ	0.051	0.053	0.050	0.016	0.011
	OC vs EW	μ	0.124	0.123	0.126		
		σ	0.055	0.057	0.054		
	OC vs MV	μ	0.081	0.079	0.083		
		σ	0.048	0.050	0.049		

Notes: See note to Table 1.

Table 3: Relative performance of portfolio strategies — LIRA Double Exponential utility

Sample	Metric		Strategy				
			BP_{med}	BP_{inf}	BP_{sup}	EW	MV
Full sample	Wealth	μ	1.019***/***	1.008***/	1.030***/***	1.005	1.010
		σ	0.005	0.004	0.006	0.003	0.003
	OC vs EW	μ	0.016	0.002	0.030		
		σ	0.007	0.008	0.008		
	OC vs MV	μ	0.013	-0.001	0.027		
		σ	0.006	0.007	0.008		
Bubble periods	Wealth	μ	1.063***/***	1.041***/***	1.076***/***	0.962	1.005
		σ	0.031	0.040	0.029	0.016	0.011
	OC vs EW	μ	0.098	0.076	0.112		
		σ	0.037	0.044	0.034		
	OC vs MV	μ	0.055	0.033	0.069		
		σ	0.030	0.041	0.026		

Notes: See note to Table 1.

Table 4: Relative performance of portfolio strategies — LIRA Uniform utility

Sample	Metric		Strategy				
			BP_{med}	BP_{inf}	BP_{sup}	EW	MV
Full sample	Wealth	μ	1.033***/***	1.032***/***	1.034***/***	1.005	1.010
		σ	0.008	0.008	0.007	0.003	0.003
	OC vs EW	μ	0.032	0.031	0.032		
		σ	0.008	0.008	0.008		
	OC vs MV	μ	0.028	0.028	0.029		
		σ	0.010	0.010	0.010		
Bubble periods	Wealth	μ	1.099***/***	1.099***/***	1.099***/***	0.962	1.005
		σ	0.030	0.030	0.030	0.016	0.011
	OC vs EW	μ	0.138	0.138	0.138		
		σ	0.039	0.039	0.039		
	OC vs MV	μ	0.094	0.094	0.094		
		σ	0.027	0.027	0.027		

Notes: See note to Table 1.

2. Empirical Application

This section provides supplementary empirical results supporting the findings of the main paper. In particular, it complements the baseline analysis by presenting additional evidence on optimal strategies, performance across the distribution of outcomes, and robustness to alternative specifications.

Subsection 2.1 reports optimal portfolio strategies for the remaining assets (*Rice*, *Soybean-oil*, and *Soybeans*). Subsection 2.2 presents results for the best and worst 10% quantiles of the performance distribution, while Subsection 2.3 focuses on investment decisions restricted to bubble periods.

2.1. Optimal portfolio strategies

This subsection reports the optimal portfolio strategies, expressed as pairs (ω^*, h^*), for each quantile of the detrended state variable and for each of the remaining speculative assets.

Consistent with the patterns documented in the main text, optimal allocations are strongly state-dependent. In particular, exposure to the speculative asset varies significantly across the distribution of the state variable, reflecting the evolving trade-off between expected returns and crash risk. As the state variable approaches the upper tail of its distribution, optimal strategies typically involve shorter horizons and reduced or even negative exposure, consistent with a rising likelihood of abrupt corrections.

Table 5: Optimal portfolio strategies (Rice)

	$q_{0.500}$	$q_{0.600}$	$q_{0.700}$	$q_{0.800}$	$q_{0.900}$	$q_{0.950}$	$q_{0.990}$	$q_{0.999}$	$q_{0..9999}$
CRRA									
$\gamma = 5$	(0.480,1)	(0.480,1)	(0.470,1)	(0.460,1)	(0.430,1)	(0.390,1) (-0.170,26)	(-0.120,26)	(-0.080,26)	(-0.060,26)
$\gamma = 10$	(0.470,1)	(0.470,1)	(0.460,1)	(0.450,1)	(0.430,1)	(0.380,1)	(0.240,1)	(-0.030,26) (0.110,1)	(-0.030,26)
LIRA									
LIRAGauss	(1,26)	(1,26)	(1,26) (-1,26)	(-1,26) (1,26)	(-1,26) (1,26)	(1,26)	(1,18)	(-1,6)	(-1,5)
LIRADExp	(0.480,1) (0,0)	(0.470,1) (0,0)	(0.470,1) (0,0)	(0.460,1) (0,0)	(0.440,1) (-0.040,26) (0,0)	(0.390,1) (-0.080,26) (0,0)	(0.240,1) (-0.070,26) (0,0)	(-0.050,26) (0.110,1) (0,0)	(-0.040,26) (0.080,1) (0,0)
LIRAUnif	(0.490,1) (0.350,26)	(0.480,1)	(0.480,1)	(0.470,1)	(0.450,1)	(0.420,1)	(-0.100,26) (0.280,1)	(-1,26)	(-1,26)
LIRAPow3	(0.480,1)	(0.470,1)	(0.470,1)	(0.460,1)	(0.440,1)	(0.400,1)	(0.240,1)	(-1,26)	(-1,26)

Notes: Optimal portfolio strategies (ω^*, h^*) conditional on the quantile of the detrended in-sample distribution in which the bubble asset is at the time of the investment. ω^* is reported in percentages of the investment and h^* in weeks.

Table 6: Optimal portfolio strategies (Soybean-oil)

	$q_{0.500}$	$q_{0.600}$	$q_{0.700}$	$q_{0.800}$	$q_{0.900}$	$q_{0.950}$	$q_{0.990}$	$q_{0.999}$	$q_{0..9999}$
CRRA									
$\gamma = 5$	(1,26)	(1,26)	(1,26)	(0.480,1) (1,26)	(0.450,1)	(1,26)	(-0.110,26)	(-0.060,26)	(-0.050,26)
$\gamma = 10$	(1,26)	(1,26)	(1,26)	(1,26)	(1,26)	(1,26)	(1,19)	(-0.030,26)	(-0.020,26)
LIRA									
LIRAGauss	(0.460,26) (0,0)	(0.410,26) (0,0)	(0.490,1) (0,0)	(0.480,1) (0,0)	(0.460,1) (-0.050,26) (0,0)	(0.420,1) (-0.140,26) (0,0)	(-0.110,26) (0.250,1) (0,0)	(-1,4)	(-1,3)
LIRADExp	(1,26)	(1,26)	(1,26)	(1,26)	(0.460,1) (0,0) (0.650,26) (0.670,25) (0.730,22) (0.750,21) (0.810,19) (0.840,18)	(0.420,1) (-0.050,26) (0,0)	(0.240,1) (-0.060,26) (0,0)	(-0.040,26) (0.080,1) (0,0)	(-0.030,26) (0.050,1) (0,0)
LIRAUnif	(0.600,26)	(0.500,26)	(0.490,1) (0.360,26)	(0.480,1)	(0.460,1)	(0.420,1)	(-0.100,26) (0.260,1)	(-1,26)	(-1,26)
LIRAPow3	(1,26)	(1,26)	(1,26)	(1,26)	(1,26)	(1,26)	(1,26) (0.250,1)	(-1,26)	(-1,26)

Notes: See note to table 5.

Table 7: Optimal portfolio strategies (Soybeans)

	$q_{0.500}$	$q_{0.600}$	$q_{0.700}$	$q_{0.800}$	$q_{0.900}$	$q_{0.950}$	$q_{0.990}$	$q_{0.999}$	$q_{0..9999}$
CRRA									
$\gamma = 5$	(0.590,26)	(0.480,26)	(0.480,1) (0.330,26)	(0.470,1)	(0.430,1)	(0.390,1) (-0.190,26)	(-0.130,26)	(-0.070,26)	(-0.050,26)
$\gamma = 10$	(0.420,26)	(0.370,26) (0.490,1)	(0.490,1)	(0.470,1)	(0.450,1)	(0.410,1)	(0.250,1)	(-0.030,26)	(-0.030,25)
LIRA									
LIRAGauss	(1,26)	(1,26) (0.500,1)	(1,26) (0.490,1) (0.450,26) (0,0)	(0.470,1) (0,0) (1,26) (-1,26)	(-1,26) (0.440,1) (-0.150,26) (0,0)	(-1,26)	(-0.210,26) (0.240,1) (0,0)	(-1,3)	(-1,2)
LIRADExp	(0.480,26) (0,0)	(0.420,26) (0.490,1) (0,0)	(0.490,1) (0.330,26) (0,0)	(0.470,1) (0,0)	(0.450,1) (-0.010,26) (0,0)	(0.410,1) (-0.090,26) (0,0)	(-0.080,26) (0.240,1) (0,0)	(-0.040,26) (0.070,1) (0,0)	(-0.030,26) (0.030,1) (0,0)
LIRAUnif	(0.660,26)	(0.530,26)	(0.490,1) (0.350,26)	(0.470,1)	(0.440,1)	(0.400,1) (-0.170,26)	(-0.140,26)	(-1,26)	(-1,26)
LIRAPow3	(0.420,26)	(0.490,1) (0.390,26)	(0.490,1)	(0.480,1)	(0.450,1)	(0.420,1)	(0.250,1)	(-1,26)	(-1,26)

Notes: See note to table 5.

2.2. BP strategies over full distribution

All out-of-sample performance results are summarized in terms of quantiles of the terminal wealth distribution across investment paths. In this subsection, we extend the analysis of the main paper by reporting results for the lower ($BP_{10\%}$) and upper ($BP_{90\%}$) quantiles of the performance distribution. We report rebalancing activity, the number of distinct strategy paths, as well as economic performance measures: terminal wealth, Sharpe ratios, modified Sharpe ratios, and opportunity costs. These results provide a more comprehensive view of the distributional properties of the BP strategies and highlight the asymmetric nature of the performance of BP strategies.

Table 8: Number of rebalancings and strategy paths – $BP_{10\%}$

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	0.000	3.000	0.000	17.000	0	3
	σ_{Rebal}	1.535	3.444	4.785	5.659	4.574	—
	$\text{Med}_{\text{Paths}}$	1	2	1	1	1	1
	σ_{Paths}	4.900	12.100	4.360	9.330	6.242	0.000
Rice	$\text{Med}_{\text{Rebal}}$	0.000	4.000	0.000	2.250	0	0
	σ_{Rebal}	0.000	4.499	5.311	4.616	—	4.542
	$\text{Med}_{\text{Paths}}$	1	3	1	1	1	1
	σ_{Paths}	0.310	10.400	4.870	24.790	0.000	1.630
Soybean oil	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	0.000	0	0
	σ_{Rebal}	1.879	5.069	4.847	0.848	2.160	—
	$\text{Med}_{\text{Paths}}$	1	1	1	1	1	1
	σ_{Paths}	9.180	24.980	3.560	0.720	1.546	0.000
Soybeans	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	0.000	0	0
	σ_{Rebal}	2.729	2.563	3.249	4.070	2.272	4.145
	$\text{Med}_{\text{Paths}}$	1	1	1	1	1	1
	σ_{Paths}	8.310	12.230	3.790	1.420	3.754	1.417

Notes: Median and standard deviation of the number of rebalancings and strategy paths for the $BP_{10\%}$ strategies across out-of-sample periods. For the equally-weighted and mean-variance portfolios, there is always one path with 26 rebalancings for each asset.

Table 9: Number of rebalancings and strategy paths – $BP_{90\%}$

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	0.000	3.000	0.000	17.000	0	3
	σ_{Rebal}	1.535	3.444	4.785	5.659	4.574	—
	$\text{Med}_{\text{Paths}}$	10	28	7	26	16	1
	σ_{Paths}	4.900	12.100	4.360	9.330	6.242	0.000
Rice	$\text{Med}_{\text{Rebal}}$	0.000	4.000	0.000	2.250	0	0
	σ_{Rebal}	0.000	4.499	5.311	4.616	—	4.542
	$\text{Med}_{\text{Paths}}$	2	28	7	45	1	2
	σ_{Paths}	0.310	10.400	4.870	24.790	0.000	1.630
Soybean oil	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	0.000	0	0
	σ_{Rebal}	1.879	5.069	4.847	0.848	2.160	—
	$\text{Med}_{\text{Paths}}$	27	28	6	1	1	1
	σ_{Paths}	9.180	24.980	3.560	0.720	1.546	0.000
Soybeans	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	0.000	0	0
	σ_{Rebal}	2.729	2.563	3.249	4.070	2.272	4.145
	$\text{Med}_{\text{Paths}}$	20	28	6	4	6	4
	σ_{Paths}	8.310	12.230	3.790	1.420	3.754	1.417

Notes: See note to Table 8.

Table 10: Out-of-sample terminal wealth and Sharpe ratios – $BP_{10\%}$

			LIRA				CRRA			
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	EW	MV
Brent	Wealth	μ	$0.950^{***/***}$	$0.970^{***/***}$	$0.990^{\cdot/**}$	0.990	$0.980^{**/***}$	$1.000^{\cdot/**}$	1.000	1.000
		σ	0.170	0.050	0.070	0.090	0.060	0.070	0.110	0.050
	Sharpe	μ	$-0.070^{***/***}$	$-0.080^{***/***}$	$-0.030^{***/*}$	$-0.010^{**/\cdot}$	$-0.040^{***/***}$	$0.020^{\cdot/**}$	0.010	-0.000
		σ	0.200	0.160	0.190	0.180	0.180	0.180	0.200	0.170
	mSharpe	μ	$-0.080^{***/***}$	$-0.100^{***/***}$	$-0.040^{***/**}$	$-0.020^{***/\cdot}$	$-0.050^{***/***}$	$0.020^{\cdot/**}$	0.010	-0.000
		σ	0.220	0.190	0.230	0.240	0.210	0.210	0.200	0.170
Rice	Wealth	μ	$0.990^{***/*}$	$0.980^{***/***}$	0.990	1.000	1.000	1.000	1.000	0.990
		σ	0.110	0.040	0.070	0.050	0.060	0.070	0.060	0.060
	Sharpe	μ	$-0.040^{***/**}$	$-0.100^{***/***}$	$-0.030^{**/\cdot}$	$-0.030^{**/\cdot}$	-0.010	-0.020	-0.000	-0.010
		σ	0.200	0.160	0.200	0.180	0.190	0.190	0.170	0.190
	mSharpe	μ	$-0.040^{***/**}$	$-0.150^{***/***}$	$-0.040^{***/**}$	$-0.040^{***/**}$	-0.020	-0.040	-0.010	-0.020
		σ	0.210	0.220	0.210	0.200	0.200	0.210	0.170	0.190
Soybean oil	Wealth	μ	$0.970^{***/***}$	$0.980^{***/***}$	0.990	0.990	1.000	1.000	1.000	0.990
		σ	0.070	0.070	0.070	0.070	0.070	0.080	0.080	0.060
	Sharpe	μ	$-0.070^{***/***}$	$-0.050^{***/***}$	-0.010	-0.020	-0.010	-0.010	-0.010	-0.010
		σ	0.190	0.200	0.200	0.200	0.200	0.200	0.210	0.190
	mSharpe	μ	$-0.080^{***/***}$	$-0.070^{***/***}$	-0.030	-0.030	-0.010	-0.010	-0.020	-0.010
		σ	0.220	0.220	0.220	0.210	0.220	0.200	0.200	0.190
Soybeans	Wealth	μ	$0.980^{***/***}$	$0.990^{\cdot/**}$	$1.000^{**/\cdot}$	$1.000^{***/*}$	0.990	1.000	0.990	0.990
		σ	0.070	0.050	0.070	0.060	0.070	0.060	0.070	0.060
	Sharpe	μ	$-0.050^{***/***}$	$-0.060^{***/***}$	-0.010	-0.010	-0.020	-0.010	-0.010	-0.020
		σ	0.210	0.180	0.190	0.180	0.190	0.190	0.190	0.180
	mSharpe	μ	$-0.070^{***/***}$	$-0.090^{***/***}$	-0.020	-0.020	-0.030	-0.030	-0.010	-0.020
		σ	0.240	0.220	0.220	0.210	0.210	0.220	0.180	0.190

Notes: Out-of-sample average (μ) and standard deviation (σ) of terminal wealth, Sharpe ratio, and modified Sharpe ratio for the $BP_{10\%}$ strategies. EW: equal-weight benchmark; MV: mean-variance benchmark. Superscripts denote significance of Wilcoxon tests vs. EW/MV: $(\cdot/*/**/***)$ at the 90%, 95%, and 99% levels; a dot $(\cdot)$ indicates non-significance; an empty slot indicates that the test could not be conducted.

Table 11: Out-of-sample terminal wealth and Sharpe ratios – $BP_{90\%}$

			LIRA				CRRA			
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	EW	MV
Brent	Wealth	μ	$1.000^{\cdot/*}$	$1.020^{***/***}$	$1.010^{**/***}$	$1.020^{***/***}$	$1.010^{**/***}$	$1.000^{\cdot/**}$	1.000	1.000
		σ	0.160	0.060	0.070	0.070	0.060	0.070	0.110	0.050
	Sharpe	μ	$0.010^{\cdot/**}$	$0.100^{***/***}$	$0.020^{\cdot/***}$	$0.080^{***/***}$	$0.030^{**/***}$	$0.020^{\cdot/**}$	0.010	-0.000
		σ	0.210	0.200	0.190	0.160	0.200	0.180	0.200	0.170
	mSharpe	μ	$0.030^{\cdot/***}$	$0.130^{***/***}$	$0.030^{\cdot/***}$	$0.100^{***/***}$	$0.040^{**/***}$	$0.020^{\cdot/**}$	0.010	-0.000
		σ	0.230	0.250	0.230	0.200	0.240	0.210	0.200	0.170
Rice	Wealth	μ	$0.990^{***/*}$	$1.030^{***/***}$	$1.000^{\cdot/*}$	$1.010^{***/***}$	1.000	$1.010^{***/***}$	1.000	0.990
		σ	0.110	0.040	0.060	0.050	0.060	0.070	0.060	0.060
	Sharpe	μ	$-0.040^{***/**}$	$0.120^{***/***}$	0.000	$0.050^{***/***}$	-0.010	0.000	-0.000	-0.010
		σ	0.200	0.150	0.190	0.170	0.190	0.180	0.170	0.190
	mSharpe	μ	$-0.040^{***/**}$	$0.150^{***/***}$	$0.000^{\cdot/*}$	$0.050^{***/***}$	-0.020	0.000	-0.010	-0.020
		σ	0.210	0.200	0.200	0.190	0.200	0.200	0.170	0.190
Soybean oil	Wealth	μ	$1.000^{*/\cdot}$	$1.020^{***/***}$	$1.010^{***/***}$	0.990	$1.000^{**/*}$	1.000	1.000	0.990
		σ	0.080	0.090	0.060	0.070	0.070	0.080	0.080	0.060
	Sharpe	μ	$0.020^{***/**}$	$0.050^{***/***}$	$0.030^{***/***}$	-0.010	$0.010^{**/**}$	-0.010	-0.010	-0.010
		σ	0.200	0.220	0.190	0.200	0.200	0.200	0.210	0.190
	mSharpe	μ	$0.030^{***/***}$	$0.060^{***/***}$	$0.040^{***/***}$	-0.010	$0.020^{**/**}$	-0.010	-0.020	-0.010
		σ	0.230	0.240	0.220	0.210	0.210	0.200	0.200	0.190
Soybeans	Wealth	μ	$1.000^{***/***}$	$1.020^{***/***}$	$1.010^{***/***}$	$1.010^{***/***}$	$1.010^{***/***}$	$1.000^{***/***}$	0.990	0.990
		σ	0.070	0.060	0.060	0.060	0.060	0.060	0.070	0.060
	Sharpe	μ	$0.030^{***/***}$	$0.070^{***/***}$	$0.040^{***/***}$	$0.020^{***/***}$	$0.040^{***/***}$	$0.010^{***/***}$	-0.010	-0.020
		σ	0.190	0.180	0.190	0.170	0.190	0.180	0.190	0.180
	mSharpe	μ	$0.040^{***/***}$	$0.080^{***/***}$	$0.040^{***/***}$	$0.020^{***/***}$	$0.040^{***/***}$	$0.020^{**/***}$	-0.010	-0.020
		σ	0.220	0.210	0.210	0.190	0.210	0.200	0.180	0.190

Notes: See note to Table 10.

Table 12: Opportunity cost – $BP_{10\%}$

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	μ	-0.050	-0.030	-0.010	-0.010	-0.020	0.000
		σ	0.180	0.110	0.120	0.100	0.120	0.110
	MV vs	μ	-0.040	-0.020	-0.010	-0.000	-0.010	0.010
		σ	0.150	0.050	0.060	0.090	0.050	0.070
Rice	EW vs	μ	-0.000	-0.020	-0.000	-0.000	-0.000	0.010
		σ	0.090	0.050	0.060	0.050	0.060	0.080
	MV vs	μ	0.000	-0.020	-0.000	0.000	0.000	0.010
		σ	0.090	0.040	0.020	0.040	0.030	0.050
Soybean oil	EW vs	μ	-0.030	-0.020	-0.010	-0.010	-0.000	-0.000
		σ	0.080	0.090	0.070	0.100	0.100	0.100
	MV vs	μ	-0.020	-0.010	-0.000	-0.000	0.000	0.000
		σ	0.060	0.040	0.040	0.030	0.040	0.040
Soybeans	EW vs	μ	-0.010	-0.010	0.000	0.010	-0.000	0.000
		σ	0.080	0.060	0.070	0.060	0.070	0.060
	MV vs	μ	-0.010	-0.010	0.000	0.010	-0.000	0.000
		σ	0.050	0.040	0.030	0.040	0.030	0.050

Notes: Out-of-sample average (μ) and standard deviation (σ) of the opportunity cost (as a fraction of initial wealth) for the $BP_{10\%}$ strategies vs. EW and MV benchmarks.

Table 13: Opportunity cost – $BP_{90\%}$

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	μ	0.000	0.020	0.000	0.020	0.000	0.000
		σ	0.150	0.120	0.120	0.080	0.120	0.110
	MV vs	μ	0.010	0.030	0.010	0.030	0.010	0.010
		σ	0.150	0.050	0.060	0.070	0.050	0.070
Rice	EW vs	μ	-0.000	0.030	0.000	0.010	-0.000	0.010
		σ	0.090	0.050	0.060	0.050	0.060	0.070
	MV vs	μ	0.000	0.030	0.010	0.020	0.000	0.010
		σ	0.090	0.040	0.030	0.030	0.030	0.050
Soybean oil	EW vs	μ	-0.000	0.020	0.010	-0.000	0.000	-0.000
		σ	0.080	0.090	0.060	0.100	0.090	0.100
	MV vs	μ	0.000	0.030	0.010	0.000	0.010	0.000
		σ	0.070	0.080	0.040	0.040	0.040	0.040
Soybeans	EW vs	μ	0.010	0.020	0.020	0.010	0.020	0.010
		σ	0.070	0.060	0.070	0.060	0.070	0.060
	MV vs	μ	0.010	0.020	0.020	0.010	0.020	0.010
		σ	0.060	0.040	0.040	0.040	0.040	0.040

Notes: See note to Table 12.

2.3. BP strategies in bubble periods

This subsection focuses on periods in which the state variable lies in the upper part of its distribution, corresponding to bubble dynamics. We consider investment decisions initiated at or above the 75% quantile threshold.

We report results for the lower ($BP_{10\%}$), median ($BP_{50\%}$), and upper ($BP_{90\%}$) strategies. Consistent with the main findings, BP strategies exhibit stronger performance in bubble periods, reflecting their ability to adapt dynamically to increases in asymmetry and crash risk. In contrast, benchmark strategies fail to adjust to these changes and therefore tend to underperform.

Table 14: Number of rebalancings and strategy paths for $BP_{10\%}$ over active periods ($q_{75\%}$ threshold)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	0	8	13.500	25	5	25
	σ_{Rebal}	1.279	8.612	10.366	9.866	8.546	9.866
	N_{Paths}	1	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	226 / 630 (35.9%)					
Rice	$\text{Med}_{\text{Rebal}}$	0	0	1	1	1	1
	σ_{Rebal}	0.000	2.436	2.380	2.380	2.380	2.380
	N_{Paths}	2	1	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	16 / 630 (2.5%)					
Soybean oil	$\text{Med}_{\text{Rebal}}$	4	4	9	0	0	0
	σ_{Rebal}	6.474	9.093	9.180	2.854	3.441	0.000
	N_{Paths}	2	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	200 / 630 (31.7%)					
Soybeans	$\text{Med}_{\text{Rebal}}$	0	8	1	11	1	11
	σ_{Rebal}	3.880	8.691	5.243	9.721	5.558	9.721
	N_{Paths}	2	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	169 / 630 (26.8%)					

Notes: Median and standard deviation of the number of rebalancings and strategy paths for the $BP_{10\%}$ strategies, computed on active out-of-sample periods only. NA stands for zero active periods. For the equally-weighted and mean-variance portfolios, there is always one path with 26 rebalancings for each asset.

Table 15: Number of rebalancings and strategy paths for $BP_{50\%}$ over active periods ($q_{75\%}$ threshold)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	0	7	15	25	3	25
	σ_{Rebal}	1.274	7.171	9.790	9.866	7.620	9.866
	N_{Paths}	1	28	1	1	3	1
	σ_{Paths}	3.240	8.210	7.480	0.000	4.720	0.000
	Active	226 / 630 (35.9%)					
Rice	$\text{Med}_{\text{Rebal}}$	0	0.500	1	1	1	1
	σ_{Rebal}	0.000	1.471	2.380	2.380	2.380	2.380
	N_{Paths}	2	3	1	1	1	1
	σ_{Paths}	0.000	2.380	0.000	0.000	0.000	0.000
	Active	16 / 630 (2.5%)					
Soybean oil	$\text{Med}_{\text{Rebal}}$	5	4	5	0	0	0
	σ_{Rebal}	5.412	6.712	8.753	0.947	2.984	0.000
	N_{Paths}	15	28	1	1	1	1
	σ_{Paths}	9.380	19.620	4.890	3.520	4.520	0.000
	Active	200 / 630 (31.7%)					
Soybeans	$\text{Med}_{\text{Rebal}}$	0	8	2	11	2	11
	σ_{Rebal}	3.988	6.767	5.402	9.721	5.416	9.721
	N_{Paths}	5	28	1	1	1	1
	σ_{Paths}	6.240	8.730	6.460	0.000	6.460	0.000
	Active	169 / 630 (26.8%)					

Notes: See note to Table 14.

Table 16: Number of rebalancings and strategy paths for $BP_{90\%}$ over active periods ($q_{75\%}$ threshold)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	0	7	19	25	3	25
	σ_{Rebal}	1.671	8.051	10.047	9.866	7.780	9.866
	N_{Paths}	1	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	226 / 630 (35.9%)					
Rice	$\text{Med}_{\text{Rebal}}$	0	0	1	1	1	1
	σ_{Rebal}	0.000	0.577	2.380	2.380	2.380	2.380
	N_{Paths}	2	1	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	16 / 630 (2.5%)					
Soybean oil	$\text{Med}_{\text{Rebal}}$	5	3	6	0	0	0
	σ_{Rebal}	6.100	7.560	8.019	1.821	4.695	0.000
	N_{Paths}	2	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	200 / 630 (31.7%)					
Soybeans	$\text{Med}_{\text{Rebal}}$	0	5	1	11	1	11
	σ_{Rebal}	4.127	6.955	5.603	9.721	5.506	9.721
	N_{Paths}	1	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	169 / 630 (26.8%)					

Notes: See note to Table 14.

Table 17: Out-of-sample terminal wealth and Sharpe ratios for $BP_{10\%}$ over active periods ($q_{75\%}$ threshold)

		LIRA				CRRA			
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	EW	MV
Brent	Med_W	0.981	$0.975^{*/***}$	0.990	$0.995^{**/***}$	$0.980^{/*}$	$0.995^{**/***}$	0.974	0.983
	$\bar{W}$	0.951	0.967	0.981	0.994	0.967	0.994	0.989	0.978
	Med_{SR}	$-0.060^{***/\cdot}$	$-0.124^{***/***}$	$-0.030^{**/\cdot}$	$-0.009^{\cdot/**}$	$-0.083^{***/***}$	$-0.011^{/**}$	-0.021	-0.045
	Med_{mSR}	$-0.066^{***/*}$	$-0.130^{***/***}$	$-0.031^{**/\cdot}$	$-0.009^{\cdot/*}$	$-0.081^{***/***}$	-0.011	-0.021	-0.045
Rice	Med_W	0.924	0.945	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	0.944	0.929
	$\bar{W}$	0.943	0.941	0.974	0.973	0.973	0.973	0.944	0.943
	Med_{SR}	-0.158	$-0.215^{**/*}$	-0.196	-0.196	-0.196	-0.196	-0.153	-0.173
	Med_{mSR}	-0.154	$-0.329^{***/***}$	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	-0.155	-0.167
Soybean oil	Med_W	$0.958^{***/***}$	$0.961^{***/***}$	$0.977^{***/*}$	$0.971^{***/**}$	$0.957^{***/***}$	$0.972^{*/}$	0.994	0.977
	$\bar{W}$	0.950	0.961	0.968	0.970	0.954	1.003	1.016	0.982
	Med_{SR}	$-0.143^{***/***}$	$-0.174^{***/***}$	$-0.103^{***/**}$	$-0.119^{***/***}$	$-0.150^{***/***}$	$-0.055^{***/}$	-0.003	-0.078
	Med_{mSR}	$-0.169^{***/***}$	$-0.172^{***/***}$	$-0.101^{***/***}$	$-0.117^{***/***}$	$-0.161^{***/***}$	$-0.055^{***/}$	-0.003	-0.077
Soybeans	Med_W	$0.944^{***/**}$	$0.958^{***/\cdot}$	$0.967^{*/\cdot}$	$0.991^{\cdot/***}$	$0.964^{*/}$	$0.991^{/***}$	0.966	0.958
	$\bar{W}$	0.944	0.956	0.962	0.981	0.962	0.981	0.979	0.956
	Med_{SR}	$-0.192^{***/***}$	$-0.194^{***/***}$	$-0.155^{***/\cdot}$	$-0.038^{\cdot/***}$	$-0.155^{***/}$	$-0.040^{/***}$	-0.074	-0.132
	Med_{mSR}	$-0.235^{***/***}$	$-0.221^{***/***}$	$-0.153^{***/\cdot}$	$-0.036^{\cdot/***}$	$-0.154^{***/}$	$-0.039^{/***}$	-0.071	-0.129

Notes: Out-of-sample median (Med) and mean ($\bar{W}$) terminal wealth, median Sharpe ratio and median modified Sharpe ratio (mSharpe) over active periods. NA stands for zero active periods. EW: equal-weight benchmark; MV: mean-variance benchmark. Superscripts denote significance of Wilcoxon tests vs. EW/MV: $(\cdot/*/**/***)$ at the 90%, 95%, and 99% levels; a dot $(\cdot)$ indicates non-significance; an empty slot indicates the test could not be conducted.

Table 18: Out-of-sample terminal wealth and Sharpe ratios for $BP_{50\%}$ over active periods ($q_{75\%}$ threshold)

		LIRA				CRRA			
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	EW	MV
Brent	Med_W	0.995	$1.000^{***/***}$	$0.996^{**/***}$	$0.995^{**/***}$	$0.998^{/*}$	$0.995^{**/***}$	0.974	0.983
	$\bar{W}$	0.960	0.996	0.994	0.994	0.984	0.994	0.989	0.978
	Med_{SR}	$-0.005^{\cdot/*}$	$0.009^{\cdot/***}$	$-0.010^{\cdot/*}$	$-0.009^{\cdot/**}$	-0.004	$-0.011^{/**}$	-0.021	-0.045
	Med_{mSR}	$-0.005^{\cdot/*}$	$0.008^{/***}$	-0.009	$-0.009^{\cdot/*}$	-0.003	-0.011	-0.021	-0.045
Rice	Med_W	0.924	$0.975^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	0.944	0.929
	$\bar{W}$	0.943	0.977	0.974	0.973	0.973	0.973	0.944	0.943
	Med_{SR}	-0.158	-0.171	-0.196	-0.196	-0.196	-0.196	-0.153	-0.173
	Med_{mSR}	-0.154	-0.221	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	-0.155	-0.167
Soybean oil	Med_W	$0.998^{\cdot/***}$	$0.991^{\cdot/*}$	$0.988^{**/\cdot}$	$0.975^{***/\cdot}$	$0.976^{***/**}$	$0.972^{*/}$	0.994	0.977
	$\bar{W}$	1.003	0.999	0.978	0.977	0.966	1.003	1.016	0.982
	Med_{SR}	$-0.013^{\cdot/***}$	$-0.048^{*/\cdot}$	$-0.046^{***/\cdot}$	$-0.088^{***/\cdot}$	$-0.097^{***/*}$	$-0.055^{***/}$	-0.003	-0.078
	Med_{mSR}	$-0.013^{\cdot/**}$	-0.049	$-0.046^{***/\cdot}$	$-0.089^{***/\cdot}$	$-0.108^{***/**}$	$-0.055^{***/}$	-0.003	-0.077
Soybeans	Med_W	$0.974^{\cdot/**}$	$0.987^{***/***}$	$0.988^{\cdot/***}$	$0.991^{\cdot/***}$	$0.986^{/***}$	$0.991^{/***}$	0.966	0.958
	$\bar{W}$	0.977	0.992	0.979	0.981	0.977	0.981	0.979	0.956
	Med_{SR}	$-0.111^{**/*}$	$-0.066^{\cdot/***}$	$-0.067^{\cdot/***}$	$-0.038^{\cdot/***}$	$-0.067^{/***}$	$-0.040^{/***}$	-0.074	-0.132
	Med_{mSR}	$-0.113^{***/\cdot}$	$-0.064^{\cdot/***}$	$-0.066^{\cdot/*}$	$-0.036^{\cdot/***}$	$-0.066^{/*}$	$-0.039^{/***}$	-0.071	-0.129

Notes: See note to Table 17.

Table 19: Out-of-sample terminal wealth and Sharpe ratios for $BP_{90\%}$ over active periods ($q_{75\%}$ threshold)

		LIRA				CRRA			
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	*EW*	*MV*
Brent	Med_W	$1.012^{**/***}$	$1.022^{***/***}$	$1.002^{***/***}$	$0.995^{**/***}$	$1.009^{***/***}$	$0.995^{**/***}$	0.974	0.983
	$\bar{W}$	0.972	1.026	1.002	0.994	0.995	0.994	0.989	0.978
	Med_{SR}	$0.055^{***/***}$	$0.140^{***/***}$	$0.013^{**/***}$	$-0.009^{\cdot/**}$	$0.062^{**/***}$	$-0.011^{/**}$	-0.021	-0.045
	Med_{mSR}	$0.061^{***/***}$	$0.160^{***/***}$	$0.013^{**/***}$	$-0.009^{\cdot/*}$	$0.062^{***/***}$	-0.011	-0.021	-0.045
Rice	Med_W	0.924	$0.995^{***/***}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	0.944	0.929
	$\bar{W}$	0.943	1.000	0.974	0.973	0.973	0.973	0.944	0.943
	Med_{SR}	-0.158	$-0.040^{**/**}$	-0.196	-0.196	-0.196	-0.196	-0.153	-0.173
	Med_{mSR}	-0.154	-0.066	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	-0.155	-0.167
Soybean oil	Med_W	$1.028^{***/***}$	$1.016^{***/***}$	0.997	$0.981^{**/\cdot}$	$0.987^{**/}$	$0.972^{*/}$	0.994	0.977
	$\bar{W}$	1.035	1.035	0.986	0.980	0.980	1.003	1.016	0.982
	Med_{SR}	$0.141^{***/***}$	$0.074^{***/***}$	$-0.014^{\cdot/*}$	$-0.052^{***/\cdot}$	$-0.045^{**/}$	$-0.055^{***/}$	-0.003	-0.078
	Med_{mSR}	$0.158^{***/***}$	$0.078^{***/***}$	$-0.015^{\cdot/*}$	$-0.055^{***/\cdot}$	$-0.045^{*/}$	$-0.055^{***/}$	-0.003	-0.077
Soybeans	Med_W	$1.005^{***/***}$	$1.013^{***/***}$	$1.000^{*/***}$	$0.991^{\cdot/***}$	$0.999^{*/***}$	$0.991^{/***}$	0.966	0.958
	$\bar{W}$	1.011	1.016	0.985	0.981	0.985	0.981	0.979	0.956
	Med_{SR}	$0.031^{***/***}$	$0.104^{***/***}$	$0.002^{\cdot/***}$	$-0.038^{\cdot/***}$	$-0.001^{/***}$	$-0.040^{/***}$	-0.074	-0.132
	Med_{mSR}	$0.038^{***/***}$	$0.110^{***/***}$	$-0.001^{\cdot/***}$	$-0.036^{\cdot/***}$	$-0.002^{/***}$	$-0.039^{/***}$	-0.071	-0.129

Notes: See note to Table 17.

Table 20: Opportunity cost for $BP_{10\%}$ over active periods ($q_{75\%}$ threshold)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	Med_{OC}	-0.001	-0.007	0.010	0.017	0.004	0.017
		σ_{OC}	0.187	0.081	0.070	0.065	–	–
	MV vs	Med_{OC}	-0.004	-0.005	0.003	0.019	-0.003	0.018
		σ_{OC}	0.126	0.035	0.041	0.048	–	–
Rice	EW vs	Med_{OC}	-0.011	0.000	0.028	0.028	0.028	0.027
		σ_{OC}	0.048	0.037	0.026	0.026	–	–
	MV vs	Med_{OC}	-0.003	-0.001	0.029	0.029	0.029	0.028
		σ_{OC}	0.012	0.018	0.038	0.038	–	–
Soybean oil	EW vs	Med_{OC}	-0.052	-0.051	-0.030	-0.030	-0.041	0.000
		σ_{OC}	0.126	0.105	0.078	0.141	–	–
	MV vs	Med_{OC}	-0.046	-0.010	-0.009	-0.005	-0.021	0.003
		σ_{OC}	0.078	0.060	0.064	0.042	–	–
Soybeans	EW vs	Med_{OC}	-0.033	-0.015	-0.010	0.001	-0.010	0.001
		σ_{OC}	0.099	0.056	0.060	0.048	–	–
	MV vs	Med_{OC}	-0.014	-0.001	-0.001	0.017	-0.001	0.017
		σ_{OC}	0.050	0.029	0.039	0.051	–	–

Notes: Median opportunity cost (Med_{OC}) and its standard deviation (σ_{OC}) for the $BP_{10\%}$ strategies vs. EW and MV benchmarks, computed on active out-of-sample periods. OC is expressed as a fraction of initial wealth. NA stands for zero active periods. No σ_{OC} is available for CRRA (denoted –).

Table 21: Opportunity cost for $BP_{50\%}$ over active periods ($q_{75\%}$ threshold)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	Med_{OC}	0.012	0.019	0.015	0.017	0.025	0.017
		σ_{OC}	0.190	0.079	0.070	0.065	–	–
	MV vs	Med_{OC}	0.006	0.020	0.022	0.019	0.008	0.018
		σ_{OC}	0.128	0.039	0.046	0.048	–	–
Rice	EW vs	Med_{OC}	-0.011	0.038	0.028	0.028	0.028	0.027
		σ_{OC}	0.048	0.027	0.026	0.026	–	–
	MV vs	Med_{OC}	-0.003	0.029	0.029	0.029	0.029	0.028
		σ_{OC}	0.012	0.040	0.038	0.038	–	–
Soybean oil	EW vs	Med_{OC}	-0.011	-0.005	-0.024	-0.024	-0.024	0.000
		σ_{OC}	0.123	0.102	0.082	0.139	–	–
	MV vs	Med_{OC}	0.016	0.019	-0.002	-0.001	-0.009	0.003
		σ_{OC}	0.087	0.075	0.067	0.046	–	–
Soybeans	EW vs	Med_{OC}	0.013	0.021	0.004	0.001	0.001	0.001
		σ_{OC}	0.108	0.052	0.050	0.048	–	–
	MV vs	Med_{OC}	0.009	0.027	0.010	0.017	0.008	0.017
		σ_{OC}	0.067	0.038	0.042	0.051	–	–

Notes: See note to Table 20.

Table 22: Opportunity cost for $BP_{90\%}$ over active periods ($q_{75\%}$ threshold)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	Med_{OC}	0.028	0.044	0.019	0.017	0.034	0.017
		σ_{OC}	0.192	0.084	0.072	0.065	–	–
	MV vs	Med_{OC}	0.017	0.047	0.032	0.019	0.018	0.018
		σ_{OC}	0.131	0.036	0.051	0.048	–	–
Rice	EW vs	Med_{OC}	-0.011	0.062	0.028	0.028	0.028	0.027
		σ_{OC}	0.048	0.013	0.026	0.026	–	–
	MV vs	Med_{OC}	-0.003	0.061	0.029	0.029	0.029	0.028
		σ_{OC}	0.012	0.035	0.038	0.038	–	–
Soybean oil	EW vs	Med_{OC}	0.037	0.049	-0.024	-0.023	-0.014	0.000
		σ_{OC}	0.129	0.108	0.086	0.140	–	–
	MV vs	Med_{OC}	0.041	0.040	-0.000	-0.001	-0.003	0.003
		σ_{OC}	0.086	0.077	0.072	0.047	–	–
Soybeans	EW vs	Med_{OC}	0.039	0.047	0.011	0.001	0.011	0.001
		σ_{OC}	0.109	0.052	0.051	0.048	–	–
	MV vs	Med_{OC}	0.036	0.052	0.016	0.017	0.016	0.017
		σ_{OC}	0.076	0.040	0.044	0.051	–	–

Notes: See note to Table 20.

3. Sensitivity Analysis

This section complements the baseline analysis by considering a restricted version of the BP framework in which only the first two conditional moments of the return distribution are taken into account to highlight the role of higher-order moments in the portfolio allocation problem. In this case, the portfolio optimization problem effectively reduces to a mean–variance framework, although it remains state-dependent through the MAR dynamics.

As in the baseline analysis, we first report the optimal portfolio strategies expressed as pairs (ω^*, h^*) for each quantile of the detrended state variable and for each of the remaining speculative assets (see Subsection 3.1). Then, in Subsection 3.2 we evaluate the performance over the full out-of-sample period, and finally we focus on investment decisions restricted to bubble periods (see Subsection 3.3). The corresponding results for rebalancing activity and performance measures are reported in Tables 27 – 35.

This emphasises the role payed by accounting for the conditional path dependence, i.e. state-dependent allocation mechanism, in portfolio analysis. The simple introduction of higher order moments provides a supplemetary refinement, namely in terms of a simpler conditional dependence structure (less paths) but does not drive by itself the performance of the proposed method.

3.1. Two-moments optimal portfolio strategies

Table 23: Optimal portfolio strategies (Brent)

	$q_{0.500}$	$q_{0.600}$	$q_{0.700}$	$q_{0.800}$	$q_{0.900}$	$q_{0.950}$	$q_{0.990}$	$q_{0.999}$	$q_{0..9999}$
CRRA									
$\gamma = 5$	(0.480,26)	(0.480,1) (0.360,26)	(0.470,1)	(0.460,1)	(0.420,1)	(0.380,1) (-0.210,26)	(-0.160,26)	(-0.100,26)	(-0.090,26)
$\gamma = 10$	(0.390,26)	(0.480,1)	(0.480,1)	(0.460,1)	(0.440,1)	(0.400,1)	(0.260,1) (-0.080,26)	(-0.060,26)	(-0.050,26)
LIRA									
LIRAGauss	(0.430,26) (0,0)	(0.480,1) (0.340,26) (0,0)	(0.470,1) (0,0)	(0.460,1) (0,0)	(0.430,1) (-0.060,26) (0,0)	(0.390,1) (-0.120,26) (0,0)	(-0.100,26) (0.240,1) (0,0)	(-0.070,26) (0.100,1) (0,0)	(-0.060,26) (0.060,1) (0,0)
LIRADExp	(0.410,26) (0.490,1) (0,0)	(0.480,1) (0.330,26) (0,0)	(0.470,1) (0,0)	(0.460,1) (0,0)	(0.440,1) (-0.040,26) (0,0)	(0.400,1) (-0.100,26) (0,0)	(-0.100,26) (0.250,1) (0,0)	(-0.070,26) (0.110,1) (0,0)	(-0.060,26) (0.070,1) (0,0)
LIRAUnif	(0.430,26)	(0.480,1) (0.340,26)	(0.470,1)	(0.460,1)	(0.430,1)	(0.390,1)	(-0.100,26) (0.240,1)	(-1,26)	(-1,26)
LIRAPow3	(0.480,1) (0.350,26)	(0.480,1)	(0.480,1)	(0.470,1)	(0.450,1)	(0.410,1)	(-1,26)	(-1,26)	(-1,26)

Notes: See note to table 5.

Table 24: Optimal portfolio strategies (Rice, first two moments only)

	$q_{0.500}$	$q_{0.600}$	$q_{0.700}$	$q_{0.800}$	$q_{0.900}$	$q_{0.950}$	$q_{0.990}$	$q_{0.999}$	$q_{0..9999}$
CRRA									
$\gamma = 5$	(0.490,1) (0.360,26)	(0.480,1)	(0.480,1)	(0.470,1)	(0.440,1)	(-0.260,26) (0.410,1)	(-0.170,26)	(-0.100,26)	(-0.080,26)
$\gamma = 10$	(0.490,1)	(0.490,1)	(0.480,1)	(0.470,1)	(0.460,1)	(0.430,1)	(0.300,1)	(-0.050,26)	(-0.050,26)
LIRA									
LIRAGauss	(0.490,1) (0.350,26) (0,0)	(0.480,1) (0,0)	(0.480,1) (0,0)	(0.470,1) (0,0)	(0.450,1) (-0.100,26) (0,0)	(0.420,1) (-0.140,26) (0,0)	(-0.100,26) (0.280,1) (0,0)	(-0.060,26) (0.130,1) (0,0)	(-0.050,26) (0.090,1) (0,0)
LIRADExp	(0.490,1) (0,0)	(0.490,1) (0,0)	(0.480,1) (0,0)	(0.470,1) (0,0)	(0.450,1) (-0.070,26) (0,0)	(0.420,1) (-0.130,26) (0,0)	(0.290,1) (-0.100,26) (0,0)	(-0.060,26) (0.140,1) (0,0)	(-0.050,26) (0.100,1) (0,0)
LIRAUnif	(0.490,1) (0.350,26)	(0.480,1)	(0.480,1)	(0.470,1)	(0.450,1)	(0.420,1)	(-0.100,26) (0.280,1)	(-1,26)	(-1,26)
LIRAPow3	(0.490,1)	(0.490,1)	(0.480,1)	(0.480,1)	(0.460,1)	(0.440,1)	(-1,26) (0.310,1)	(-1,26)	(-1,26)

Notes: See note to table 5.

Table 25: Optimal portfolio strategies (Soybean-oil)

	$q_{0.500}$	$q_{0.600}$	$q_{0.700}$	$q_{0.800}$	$q_{0.900}$	$q_{0.950}$	$q_{0.990}$	$q_{0.999}$	$q_{0..9999}$
CRRA									
$\gamma = 5$	(0.680,26)	(0.530,26)	(0.490,1) (0.370,26)	(0.480,1)	(0.460,1)	(0.420,1) (-0.200,26)	(-0.160,26)	(-0.090,26)	(-0.070,26)
$\gamma = 10$	(0.540,26)	(0.460,26)	(0.490,1)	(0.480,1)	(0.470,1)	(0.430,1)	(0.280,1)	(-0.050,26)	(-0.040,26)
LIRA									
LIRAGauss	(0.590,26) (0,0)	(0.490,26) (0,0)	(0.490,1) (0.360,26) (0,0)	(0.480,1) (0,0)	(0.460,1) (-0.010,26) (0,0)	(0.420,1) (-0.110,26) (0,0)	(-0.100,26) (0.260,1) (0,0)	(-0.060,26) (0.090,1) (0,0)	(-1,26)
LIRADExp	(0.570,26) (0,0)	(0.470,26) (0.500,1) (0,0)	(0.490,1) (0.360,26) (0,0)	(0.480,1) (0,0)	(0.460,1) (0,0)	(0.430,1) (-0.080,26) (0,0)	(-0.100,26) (0.270,1) (0,0)	(-0.060,26) (0.090,1) (0,0)	(-0.050,26) (0.050,1) (0,0)
LIRAUnif	(0.600,26)	(0.500,26)	(0.490,1) (0.360,26)	(0.480,1)	(0.460,1)	(0.420,1)	(-0.100,26) (0.260,1)	(-1,26)	(-1,26)
LIRAPow3	(0.480,26)	(0.420,26) (0.500,1)	(0.490,1)	(0.490,1)	(0.470,1)	(0.440,1)	(-1,26)	(-1,26)	(-1,26)

Notes: See note to table 5.

Table 26: Optimal portfolio strategies (Soybeans, first two moments only)

	$q_{0.500}$	$q_{0.600}$	$q_{0.700}$	$q_{0.800}$	$q_{0.900}$	$q_{0.950}$	$q_{0.990}$	$q_{0.999}$	$q_{0..9999}$
CRRA									
$\gamma = 5$	(0.760,26)	(0.560,26)	(0.490,1) (0.350,26)	(0.470,1)	(0.440,1)	(-0.310,26) (0.390,1)	(-0.260,26)	(-0.130,26)	(-0.100,26)
$\gamma = 10$	(0.590,26)	(0.480,26)	(0.490,1)	(0.480,1)	(0.450,1)	(0.420,1)	(-0.140,26)	(-1,26)	(-1,26)
LIRA									
LIRAGauss	(0.650,26) (0,0)	(0.520,26) (0.500,1) (0,0)	(0.490,1) (0.350,26) (0,0)	(0.470,1) (0,0)	(0.440,1) (-0.080,26) (0,0)	(0.410,1) (-0.170,26) (0,0)	(-0.140,26) (0.250,1) (0,0)	(-1,26)	(-1,26)
LIRADExp	(0.630,26) (0,0)	(0.500,26) (0.500,1) (0,0)	(0.490,1) (0.360,26) (0,0)	(0.470,1) (0,0)	(0.450,1) (-0.040,26) (0,0)	(0.410,1) (-0.150,26) (0,0)	(-0.150,26) (0.260,1) (0,0)	(-0.090,26) (-1,26) (0.070,1) (0,0)	(-1,26)
LIRAUnif	(0.660,26)	(0.530,26)	(0.490,1) (0.350,26)	(0.470,1)	(0.440,1)	(0.400,1) (-0.170,26)	(-0.140,26)	(-1,26)	(-1,26)
LIRAPow3	(0.520,26)	(0.450,26) (0.490,1)	(0.490,1) (0.360,26)	(0.480,1)	(0.460,1)	(0.430,1)	(-1,26)	(-1,26)	(-1,26)

Notes: See note to table 5.

3.2. Two-moments BP strategies over full distribution

Table 27: Number of rebalancings and strategy paths – $BP_{10\%}$ (first two moments only)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	0.000	1.000	0.000	1.000	0	0
	σ_{Rebal}	4.355	4.168	4.944	4.231	5.741	4.214
	$\text{Med}_{\text{Paths}}$	1	1	1	1	1	1
	σ_{Paths}	12.920	12.620	5.100	1.990	3.561	5.019
Rice	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	2.000	0	0
	σ_{Rebal}	2.242	2.723	4.357	5.641	4.357	4.579
	$\text{Med}_{\text{Paths}}$	1	1	1	1	1	1
	σ_{Paths}	10.640	11.370	1.680	9.730	1.678	1.214
Soybean oil	$\text{Med}_{\text{Rebal}}$	0.000	0.500	0.000	0.000	0	0
	σ_{Rebal}	2.866	4.141	4.710	4.156	4.421	—
	$\text{Med}_{\text{Paths}}$	1	1	1	1	1	1
	σ_{Paths}	11.560	12.450	5.810	2.690	5.408	0.000
Soybeans	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	0.000	0	0
	σ_{Rebal}	2.682	2.663	3.849	3.875	3.845	—
	$\text{Med}_{\text{Paths}}$	1	1	1	1	1	1
	σ_{Paths}	10.120	11.420	4.090	4.080	4.086	0.000

Notes: Median and standard deviation of the number of rebalancings and strategy paths for the $BP_{50\%}$ strategies across out-of-sample periods. For the equally-weighted and mean-variance portfolios, there is always one path with 26 rebalancings for each asset.

Table 28: Number of rebalancings and strategy paths – $BP_{50\%}$ (first two moments only)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	0.000	1.000	0.000	1.000	0	0
	σ_{Rebal}	4.355	4.168	4.944	4.231	5.741	4.214
	$\text{Med}_{\text{Paths}}$	18	27	1	1	1	1
	σ_{Paths}	12.920	12.620	5.100	1.990	3.561	5.019
Rice	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	2.000	0	0
	σ_{Rebal}	2.242	2.723	4.357	5.641	4.357	4.579
	$\text{Med}_{\text{Paths}}$	2	3	1	1	1	1
	σ_{Paths}	10.640	11.370	1.680	9.730	1.678	1.214
Soybean oil	$\text{Med}_{\text{Rebal}}$	0.000	0.500	0.000	0.000	0	0
	σ_{Rebal}	2.866	4.141	4.710	4.156	4.421	—
	$\text{Med}_{\text{Paths}}$	2	24	1	1	1	1
	σ_{Paths}	11.560	12.450	5.810	2.690	5.408	0.000
Soybeans	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	0.000	0	0
	σ_{Rebal}	2.682	2.663	3.849	3.875	3.845	—
	$\text{Med}_{\text{Paths}}$	2	2	1	1	1	1
	σ_{Paths}	10.120	11.420	4.090	4.080	4.086	0.000

Notes: See note to table 27.

Table 29: Number of rebalancings and strategy paths – $BP_{90\%}$ (first two moments only)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	0.000	1.000	0.000	1.000	0	0
	σ_{Rebal}	4.355	4.168	4.944	4.231	5.741	4.214
	$\text{Med}_{\text{Paths}}$	28	28	9	3	8	7.100
	σ_{Paths}	12.920	12.620	5.100	1.990	3.561	5.019
Rice	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	2.000	0	0
	σ_{Rebal}	2.242	2.723	4.357	5.641	4.357	4.579
	$\text{Med}_{\text{Paths}}$	27	27	3	27	3.100	2
	σ_{Paths}	10.640	11.370	1.680	9.730	1.678	1.214
Soybean oil	$\text{Med}_{\text{Rebal}}$	0.000	0.500	0.000	0.000	0	0
	σ_{Rebal}	2.866	4.141	4.710	4.156	4.421	—
	$\text{Med}_{\text{Paths}}$	28	28	12	6	11	1
	σ_{Paths}	11.560	12.450	5.810	2.690	5.408	0.000
Soybeans	$\text{Med}_{\text{Rebal}}$	0.000	0.000	0.000	0.000	0	0
	σ_{Rebal}	2.682	2.663	3.849	3.875	3.845	—
	$\text{Med}_{\text{Paths}}$	28	28	6	7	6	1
	σ_{Paths}	10.120	11.420	4.090	4.080	4.086	0.000

Notes: See note to table 27.

Table 30: Out-of-sample terminal wealth and Sharpe ratios – $BP_{10\%}$ (first two moments only)

			LIRA				CRRA			
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	*EW*	*MV*
Brent	Wealth	μ	$0.980^{***/***}$	$0.980^{***/***}$	$0.990^{*/**}$	0.990	$0.980^{***/***}$	0.990	1.000	1.000
		σ	0.060	0.060	0.060	0.070	0.060	0.060	0.110	0.050
	Sharpe	μ	$-0.060^{***/***}$	$-0.060^{***/***}$	$-0.030^{***/**}$	$-0.010^{**/\cdot}$	$-0.040^{***/***}$	-0.010	0.010	-0.000
		σ	0.170	0.170	0.180	0.180	0.180	0.170	0.200	0.170
	mSharpe	μ	$-0.080^{***/***}$	$-0.080^{***/***}$	$-0.040^{***/**}$	$-0.030^{***/\cdot}$	$-0.060^{***/***}$	$-0.010^{*/\cdot}$	0.010	-0.000
		σ	0.200	0.200	0.210	0.220	0.210	0.180	0.200	0.170
Rice	Wealth	μ	$0.990^{*/\cdot}$	$0.980^{***/***}$	1.000	1.000	1.000	1.000	1.000	0.990
		σ	0.060	0.050	0.060	0.050	0.060	0.080	0.060	0.060
	Sharpe	μ	$-0.050^{***/***}$	$-0.080^{***/***}$	-0.020	$-0.030^{***/*}$	-0.020	-0.020	-0.000	-0.010
		σ	0.200	0.180	0.190	0.180	0.190	0.190	0.170	0.190
	mSharpe	μ	$-0.070^{***/***}$	$-0.110^{***/***}$	$-0.040^{*/\cdot}$	$-0.060^{***/***}$	$-0.040^{*/\cdot}$	-0.030	-0.010	-0.020
		σ	0.230	0.220	0.210	0.220	0.210	0.200	0.170	0.190
Soybean oil	Wealth	μ	$0.980^{*/***}$	$0.980^{***/***}$	$0.990^{\cdot/**}$	0.990	$0.980^{\cdot/***}$	$1.000^{*/\cdot}$	1.000	0.990
		σ	0.060	0.050	0.060	0.070	0.060	0.060	0.080	0.060
	Sharpe	μ	$-0.060^{***/***}$	$-0.080^{***/***}$	$-0.040^{**/***}$	$-0.030^{*/**}$	$-0.040^{**/***}$	-0.000	-0.010	-0.010
		σ	0.190	0.170	0.190	0.210	0.190	0.200	0.210	0.190
	mSharpe	μ	$-0.080^{***/***}$	$-0.120^{***/***}$	$-0.060^{***/***}$	$-0.040^{**/***}$	$-0.070^{***/***}$	-0.010	-0.020	-0.010
		σ	0.220	0.220	0.230	0.250	0.230	0.230	0.200	0.190
Soybeans	Wealth	μ	$0.990^{\cdot/**}$	$0.990^{\cdot/**}$	0.990	0.990	0.990	$1.000^{**/\cdot}$	0.990	0.990
		σ	0.060	0.060	0.060	0.060	0.060	0.060	0.070	0.060
	Sharpe	μ	$-0.060^{***/***}$	$-0.060^{***/***}$	-0.020	$-0.030^{*/\cdot}$	-0.020	$0.000^{\cdot/*}$	-0.010	-0.020
		σ	0.200	0.190	0.200	0.200	0.200	0.190	0.190	0.180
	mSharpe	μ	$-0.090^{***/***}$	$-0.090^{***/***}$	-0.040	$-0.050^{***/**}$	-0.040	-0.000	-0.010	-0.020
		σ	0.230	0.230	0.220	0.230	0.220	0.210	0.180	0.190

Notes: Out-of-sample average (μ) and standard deviation (σ) of terminal wealth, Sharpe ratio, and modified Sharpe ratio (mSharpe) for the $BP_{10\%}$ strategies. EW: equal-weight benchmark; MV: mean-variance benchmark. For μ rows, superscripts denote significance of Wilcoxon tests vs. EW/MV: $(\cdot/*/**/***)$ at the 90%, 95%, and 99% levels; a dot $(\cdot)$ indicates non-significance.

Table 31: Out-of-sample terminal wealth and Sharpe ratios – $BP_{50\%}$ (first two moments only)

			LIRA				CRRA			
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	*EW*	*MV*
Brent	Wealth	μ	1.000	1.010$^{**/***}$	1.000	0.990	0.990	1.000	1.000	1.000
		σ	0.060	0.050	0.060	0.070	0.060	0.060	0.110	0.050
	Sharpe	μ	0.010$^{\cdot/**}$	0.030$^{*/***}$	-0.000	0.000	-0.000	0.010$^{\cdot/*}$	0.010	-0.000
		σ	0.170	0.160	0.180	0.170	0.180	0.170	0.200	0.170
	mSharpe	μ	0.010$^{\cdot/**}$	0.030$^{*/***}$	-0.010	0.000	-0.000	0.000	0.010	-0.000
		σ	0.180	0.180	0.190	0.200	0.190	0.190	0.200	0.170
Rice	Wealth	μ	1.000$^{*/*}$	1.000$^{*/\cdot}$	1.000	1.010$^{***/***}$	1.000	1.000	1.000	0.990
		σ	0.060	0.050	0.060	0.050	0.060	0.080	0.060	0.060
	Sharpe	μ	-0.000	-0.010	-0.010	0.010$^{**/**}$	-0.010	-0.010	-0.000	-0.010
		σ	0.180	0.180	0.190	0.160	0.190	0.180	0.170	0.190
	mSharpe	μ	-0.010	-0.010	-0.020	0.000$^{**/**}$	-0.020	-0.020	-0.010	-0.020
		σ	0.190	0.190	0.200	0.180	0.200	0.190	0.170	0.190
Soybean oil	Wealth	μ	1.000$^{***/**}$	1.000$^{**/\cdot}$	1.000	0.990	1.000	1.000$^{*/\cdot}$	1.000	0.990
		σ	0.060	0.050	0.060	0.070	0.060	0.060	0.080	0.060
	Sharpe	μ	0.010$^{***/**}$	0.010$^{*/\cdot}$	-0.000	-0.010	-0.000	-0.000	-0.010	-0.010
		σ	0.170	0.170	0.180	0.200	0.180	0.200	0.210	0.190
	mSharpe	μ	0.010$^{**/*}$	0.000	-0.010	-0.020	-0.010	-0.010	-0.020	-0.010
		σ	0.190	0.190	0.200	0.230	0.200	0.230	0.200	0.190
Soybeans	Wealth	μ	1.000$^{***/**}$	1.000$^{***/***}$	1.000$^{***/**}$	1.000$^{**/\cdot}$	1.000$^{***/**}$	1.000$^{**/\cdot}$	0.990	0.990
		σ	0.060	0.050	0.060	0.060	0.060	0.060	0.070	0.060
	Sharpe	μ	0.010$^{**/**}$	0.010$^{**/**}$	0.010$^{**/**}$	-0.000	0.010$^{*/**}$	0.000$^{\cdot/*}$	-0.010	-0.020
		σ	0.180	0.180	0.190	0.190	0.190	0.190	0.190	0.180
	mSharpe	μ	0.010$^{**/**}$	0.000$^{*/**}$	0.000$^{\cdot/**}$	-0.010	0.000$^{\cdot/**}$	-0.000	-0.010	-0.020
		σ	0.200	0.190	0.200	0.210	0.200	0.210	0.180	0.190

Notes: See note to table 30.

Table 32: Out-of-sample terminal wealth and Sharpe ratios – $BP_{90\%}$ (first two moments only)

			LIRA				CRRA			
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	*EW*	*MV*
Brent	Wealth	μ	1.020$^{***/***}$	1.030$^{***/***}$	1.000$^{**/***}$	1.000$^{\cdot/**}$	1.000$^{*/***}$	1.000$^{\cdot/**}$	1.000	1.000
		σ	0.060	0.060	0.060	0.070	0.060	0.060	0.110	0.050
	Sharpe	μ	0.080$^{***/***}$	0.110$^{***/***}$	0.030$^{\cdot/***}$	0.030$^{\cdot/***}$	0.030$^{**/***}$	0.020$^{\cdot/***}$	0.010	-0.000
		σ	0.180	0.180	0.180	0.180	0.190	0.180	0.200	0.170
	mSharpe	μ	0.100$^{***/***}$	0.140$^{***/***}$	0.040$^{*/***}$	0.030$^{\cdot/***}$	0.040$^{**/***}$	0.030$^{\cdot/***}$	0.010	-0.000
		σ	0.220	0.220	0.210	0.200	0.220	0.210	0.200	0.170
Rice	Wealth	μ	1.010$^{***/***}$	1.010$^{***/***}$	1.000$^{*/*}$	1.010$^{***/***}$	1.000$^{*/*}$	1.000$^{*/*}$	1.000	0.990
		σ	0.060	0.060	0.060	0.050	0.060	0.080	0.060	0.060
	Sharpe	μ	0.040$^{***/***}$	0.060$^{***/***}$	-0.000	0.060$^{***/***}$	-0.000	-0.010	-0.000	-0.010
		σ	0.190	0.190	0.180	0.160	0.180	0.180	0.170	0.190
	mSharpe	μ	0.050$^{***/***}$	0.080$^{***/***}$	-0.010	0.070$^{***/***}$	-0.010	-0.010	-0.010	-0.020
		σ	0.220	0.230	0.190	0.190	0.190	0.190	0.170	0.190
Soybean oil	Wealth	μ	1.020$^{***/***}$	1.020$^{***/***}$	1.010$^{***/***}$	1.000$^{**/\cdot}$	1.010$^{***/***}$	1.000$^{*/\cdot}$	1.000	0.990
		σ	0.060	0.050	0.060	0.070	0.060	0.060	0.080	0.060
	Sharpe	μ	0.080$^{***/***}$	0.090$^{***/***}$	0.040$^{***/***}$	0.010$^{**/*}$	0.040$^{***/***}$	-0.000	-0.010	-0.010
		σ	0.180	0.180	0.180	0.200	0.180	0.200	0.210	0.190
	mSharpe	μ	0.100$^{***/***}$	0.110$^{***/***}$	0.060$^{***/***}$	0.020$^{**/**}$	0.050$^{***/***}$	-0.010	-0.020	-0.010
		σ	0.210	0.220	0.210	0.220	0.210	0.230	0.200	0.190
Soybeans	Wealth	μ	1.010$^{***/***}$	1.010$^{***/***}$	1.010$^{***/***}$	1.000$^{***/***}$	1.010$^{***/***}$	1.000$^{**/\cdot}$	0.990	0.990
		σ	0.060	0.060	0.060	0.060	0.060	0.060	0.070	0.060
	Sharpe	μ	0.070$^{***/***}$	0.070$^{***/***}$	0.030$^{***/***}$	0.030$^{***/***}$	0.030$^{***/***}$	0.000$^{\cdot/*}$	-0.010	-0.020
		σ	0.190	0.190	0.190	0.190	0.190	0.190	0.190	0.180
	mSharpe	μ	0.080$^{***/***}$	0.080$^{***/***}$	0.040$^{***/***}$	0.040$^{***/***}$	0.040$^{***/***}$	-0.000	-0.010	-0.020
		σ	0.220	0.210	0.220	0.220	0.220	0.210	0.180	0.190

Notes: See note to table 30.

Table 33: Opportunity cost – $BP_{10\%}$ (first two moments only)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	μ	-0.020	-0.020	-0.020	-0.010	-0.020	-0.010
		σ	0.120	0.110	0.120	0.120	0.120	0.110
	MV vs	μ	-0.010	-0.010	-0.010	-0.010	-0.010	-0.000
		σ	0.050	0.050	0.060	0.070	0.060	0.050
Rice	EW vs	μ	-0.010	-0.010	-0.000	-0.000	-0.000	0.000
		σ	0.060	0.050	0.060	0.050	0.060	0.080
	MV vs	μ	-0.000	-0.010	0.000	0.000	0.000	0.010
		σ	0.030	0.040	0.030	0.040	0.030	0.060
Soybean oil	EW vs	μ	-0.010	-0.020	-0.010	-0.010	-0.010	-0.000
		σ	0.080	0.070	0.070	0.070	0.080	0.060
	MV vs	μ	-0.010	-0.020	-0.010	-0.000	-0.010	0.000
		σ	0.040	0.040	0.040	0.050	0.040	0.040
Soybeans	EW vs	μ	-0.010	-0.010	-0.000	-0.000	-0.000	0.010
		σ	0.070	0.060	0.070	0.060	0.070	0.060
	MV vs	μ	-0.010	-0.010	-0.000	-0.000	-0.000	0.010
		σ	0.030	0.030	0.030	0.040	0.030	0.040

Notes: Opportunity cost (as a fraction of initial wealth) for the $BP_{10\%}$ strategies vs. EW and MV benchmarks. Out-of-sample average (μ) and standard deviation (σ).

Table 34: Opportunity cost – $BP_{50\%}$ (first two moments only)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	μ	-0.000	0.000	-0.010	-0.010	-0.010	-0.000
		σ	0.120	0.110	0.110	0.110	0.120	0.110
	MV vs	μ	0.000	0.010	-0.000	-0.000	-0.000	0.010
		σ	0.050	0.050	0.050	0.060	0.050	0.050
Rice	EW vs	μ	0.000	0.000	0.000	0.010	0.000	0.010
		σ	0.060	0.050	0.060	0.050	0.060	0.080
	MV vs	μ	0.010	0.010	0.010	0.010	0.010	0.010
		σ	0.030	0.040	0.030	0.040	0.030	0.060
Soybean oil	EW vs	μ	0.000	0.000	-0.000	-0.010	-0.000	-0.000
		σ	0.070	0.070	0.070	0.070	0.080	0.060
	MV vs	μ	0.010	0.010	0.000	-0.000	0.000	0.000
		σ	0.040	0.040	0.040	0.050	0.040	0.040
Soybeans	EW vs	μ	0.010	0.010	0.010	0.000	0.010	0.010
		σ	0.060	0.060	0.060	0.060	0.060	0.060
	MV vs	μ	0.010	0.010	0.010	0.000	0.010	0.010
		σ	0.040	0.040	0.030	0.040	0.030	0.040

Notes: See note to table 33.

Table 35: Opportunity cost – $BP_{90\%}$ (first two moments only)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	μ	0.010	0.020	0.000	0.000	0.000	0.000
		σ	0.120	0.110	0.120	0.110	0.120	0.110
	MV vs	μ	0.020	0.030	0.010	0.010	0.010	0.010
		σ	0.050	0.050	0.050	0.060	0.050	0.060
Rice	EW vs	μ	0.010	0.020	0.000	0.020	0.000	0.010
		σ	0.060	0.060	0.060	0.050	0.060	0.080
	MV vs	μ	0.020	0.020	0.010	0.020	0.010	0.010
		σ	0.040	0.040	0.030	0.040	0.030	0.060
Soybean oil	EW vs	μ	0.020	0.020	0.010	0.000	0.010	-0.000
		σ	0.070	0.070	0.070	0.070	0.080	0.060
	MV vs	μ	0.020	0.030	0.010	0.010	0.010	0.000
		σ	0.040	0.040	0.040	0.040	0.050	0.040
Soybeans	EW vs	μ	0.020	0.020	0.010	0.010	0.010	0.010
		σ	0.060	0.060	0.060	0.060	0.060	0.060
	MV vs	μ	0.020	0.020	0.010	0.010	0.010	0.010
		σ	0.040	0.040	0.030	0.030	0.030	0.040

Notes: See note to table 33.

3.3. Two-moments BP strategies in bubble periods

Table 36: Number of rebalancings and strategy paths for $BP_{10\%}$ over active periods ($q_{75\%}$ threshold, first two moments only)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	7	8	13.500	8.500	4.500	13.500
	σ_{Rebal}	8.468	8.550	10.366	10.935	8.663	10.301
	N_{Paths}	3	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	226 / 630 (35.9%)					
Rice	$\text{Med}_{\text{Rebal}}$	0	0	1	1	1	1
	σ_{Rebal}	2.436	2.436	2.380	2.380	2.380	2.380
	N_{Paths}	1	1	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	16 / 630 (2.5%)					
Soybean oil	$\text{Med}_{\text{Rebal}}$	3	6	9	2	0	10.500
	σ_{Rebal}	7.465	8.733	9.180	8.302	7.494	9.868
	N_{Paths}	2	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	200 / 630 (31.7%)					
Soybeans	$\text{Med}_{\text{Rebal}}$	2	8	1	3	1	3
	σ_{Rebal}	6.035	8.816	5.243	8.531	5.548	8.531
	N_{Paths}	2	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	169 / 630 (26.8%)					

Notes: Median and standard deviation of the number of rebalancings and strategy paths for the $BP_{10\%}$ strategies, computed on active out-of-sample periods only. NA stands for zero active periods. For the equally-weighted and mean-variance portfolios, there is always one path with 26 rebalancings for each asset.

Table 37: Number of rebalancings and strategy paths for $BP_{50\%}$ over active periods ($q_{75\%}$ threshold, first two moments only)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	7	7	15	8.500	3	16
	σ_{Rebal}	7.571	7.669	9.790	10.935	7.277	9.777
	N_{Paths}	28	28	1	1	3	1
	σ_{Paths}	8.210	8.210	7.480	0.000	4.720	7.480
	Active	226 / 630 (35.9%)					
Rice	$\text{Med}_{\text{Rebal}}$	0	0	1	1	1	1
	σ_{Rebal}	1.436	1.436	2.380	2.380	2.380	2.380
	N_{Paths}	3	3	1	1	1	1
	σ_{Paths}	2.380	2.380	0.000	0.000	0.000	0.000
	Active	16 / 630 (2.5%)					
Soybean oil	$\text{Med}_{\text{Rebal}}$	5	7	5	2	0	10.500
	σ_{Rebal}	6.465	6.306	8.753	8.302	7.032	9.868
	N_{Paths}	18	28	1	1	1	1
	σ_{Paths}	9.690	8.360	4.890	0.000	2.280	0.000
	Active	200 / 630 (31.7%)					
Soybeans	$\text{Med}_{\text{Rebal}}$	3	8	2	3	1	3
	σ_{Rebal}	6.484	6.731	5.402	8.531	5.483	8.531
	N_{Paths}	14	28	1	1	1	1
	σ_{Paths}	10.000	8.740	6.460	0.000	6.460	0.000
	Active	169 / 630 (26.8%)					

Notes: See note to table 36.

Table 38: Number of rebalancings and strategy paths for $BP_{90\%}$ over active periods ($q_{75\%}$ threshold, first two moments only)

		LIRA				CRRA	
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	$\text{Med}_{\text{Rebal}}$	9	9	19	8.500	3	19
	σ_{Rebal}	8.158	8.224	10.047	10.935	7.652	9.945
	N_{Paths}	3	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	226 / 630 (35.9%)					
Rice	$\text{Med}_{\text{Rebal}}$	0	0	1	1	1	1
	σ_{Rebal}	0.577	0.577	2.380	2.380	2.380	2.380
	N_{Paths}	1	1	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	16 / 630 (2.5%)					
Soybean oil	$\text{Med}_{\text{Rebal}}$	4	7	6	2	1	10.500
	σ_{Rebal}	6.754	8.197	8.019	8.302	7.202	9.868
	N_{Paths}	2	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	200 / 630 (31.7%)					
Soybeans	$\text{Med}_{\text{Rebal}}$	2	5	1	3	1	3
	σ_{Rebal}	5.696	6.345	5.603	8.531	5.533	8.531
	N_{Paths}	2	3	1	1	1	1
	σ_{Paths}	—	—	—	—	—	—
	Active	169 / 630 (26.8%)					

Notes: See note to table 36.

Table 39: Out-of-sample terminal wealth and Sharpe ratios – $BP_{10\%}$ over active periods ($q_{75\%}$, first two moments only))

		LIRA				CRRA			
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	EW	MV
Brent	Med_W	$0.974^{*/***}$	$0.974^{*/***}$	0.990	0.989	$0.979^{/*}$	0.989	0.974	0.983
	$\bar{W}$	0.967	0.967	0.981	0.978	0.967	0.981	0.989	0.978
	Med_{SR}	$-0.123^{***/***}$	$-0.120^{***/***}$	$-0.030^{**/\cdot}$	$-0.026^{**/\cdot}$	$-0.082^{***/***}$	$-0.032^{**/}$	-0.021	-0.045
	Med_{mSR}	$-0.126^{***/***}$	$-0.123^{***/***}$	$-0.031^{**/\cdot}$	$-0.026^{**/\cdot}$	$-0.083^{***/***}$	$-0.032^{**/}$	-0.021	-0.045
Rice	Med_W	0.945	0.945	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	0.944	0.929
	$\bar{W}$	0.941	0.941	0.974	0.974	0.974	0.974	0.944	0.943
	Med_{SR}	$-0.215^{**/*}$	$-0.215^{**/*}$	-0.196	-0.196	-0.196	-0.196	-0.153	-0.173
	Med_{mSR}	$-0.329^{***/***}$	$-0.329^{***/***}$	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	-0.155	-0.167
Soybean oil	Med_W	$0.967^{***/***}$	$0.966^{***/***}$	$0.977^{***/*}$	$0.978^{***/\cdot}$	$0.958^{***/***}$	$0.987^{**/}$	0.994	0.977
	$\bar{W}$	0.950	0.956	0.968	0.973	0.957	0.977	1.016	0.982
	Med_{SR}	$-0.170^{***/***}$	$-0.172^{***/***}$	$-0.103^{***/**}$	$-0.081^{***/\cdot}$	$-0.132^{***/***}$	$-0.031^{***/}$	-0.003	-0.078
	Med_{mSR}	$-0.205^{***/***}$	$-0.187^{***/***}$	$-0.101^{***/***}$	$-0.081^{***/\cdot}$	$-0.135^{***/***}$	$-0.032^{**/}$	-0.003	-0.077
Soybeans	Med_W	$0.955^{***/\cdot}$	$0.957^{***/\cdot}$	$0.967^{*/\cdot}$	$0.980^{\cdot/***}$	$0.964^{*/}$	$0.980^{/***}$	0.966	0.958
	$\bar{W}$	0.948	0.956	0.962	0.975	0.962	0.975	0.979	0.956
	Med_{SR}	$-0.212^{***/***}$	$-0.193^{***/***}$	$-0.155^{***/\cdot}$	$-0.065^{\cdot/***}$	$-0.155^{***/}$	$-0.072^{/***}$	-0.074	-0.132
	Med_{mSR}	$-0.261^{***/***}$	$-0.229^{***/***}$	$-0.153^{***/\cdot}$	$-0.065^{\cdot/*}$	$-0.154^{***/}$	$-0.071^{/*}$	-0.071	-0.129

Notes: Out-of-sample average (μ) and standard deviation (σ) of terminal wealth, Sharpe ratio, and modified Sharpe ratio (mSharpe) for the $BP_{10\%}$ strategies. EW: equal-weight benchmark; MV: mean-variance benchmark. For μ rows, superscripts denote significance of Wilcoxon tests vs. EW/MV: $(\cdot/*/**/***)$ at the 90%, 95%, and 99% levels; a dot $(\cdot)$ indicates non-significance; an empty slot indicates that the test could not be conducted. NA stands for zero active periods.

Table 40: Out-of-sample terminal wealth and Sharpe ratios – $BP_{50\%}$ over active periods ($q_{75\%}$, first two moments only)

		LIRA				CRRA			
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	EW	MV
Brent	Med_W	$1.000^{***/***}$	$1.002^{***/***}$	$0.996^{**/***}$	0.989	$1.000^{*/**}$	$0.996^{***/***}$	0.974	0.983
	$\bar{W}$	0.997	0.997	0.994	0.978	0.986	0.994	0.989	0.978
	Med_{SR}	$0.013^{\cdot/***}$	$0.013^{\cdot/***}$	$-0.010^{\cdot/*}$	$-0.026^{**/\cdot}$	-0.002	$-0.011^{/*}$	-0.021	-0.045
	Med_{mSR}	$0.012^{\cdot/***}$	$0.009^{\cdot/***}$	-0.009	$-0.026^{**/\cdot}$	0.002	-0.010	-0.021	-0.045
Rice	Med_W	$0.979^{***/**}$	$0.979^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	0.944	0.929
	$\bar{W}$	0.982	0.982	0.974	0.974	0.974	0.974	0.944	0.943
	Med_{SR}	-0.171	-0.171	-0.196	-0.196	-0.196	-0.196	-0.153	-0.173
	Med_{mSR}	-0.215	-0.215	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	-0.155	-0.167
Soybean oil	Med_W	$0.996^{\cdot/***}$	$0.992^{\cdot/***}$	$0.988^{**/\cdot}$	$0.978^{***/\cdot}$	$0.980^{***/}$	$0.987^{**/}$	0.994	0.977
	$\bar{W}$	0.995	0.997	0.978	0.973	0.972	0.977	1.016	0.982
	Med_{SR}	$-0.017^{*/*}$	$-0.042^{\cdot/**}$	$-0.046^{***/\cdot}$	$-0.081^{***/\cdot}$	$-0.068^{***/}$	$-0.031^{***/}$	-0.003	-0.078
	Med_{mSR}	$-0.016^{*/\cdot}$	$-0.043^{\cdot/*}$	$-0.046^{***/\cdot}$	$-0.081^{***/\cdot}$	$-0.068^{***/}$	$-0.032^{**/}$	-0.003	-0.077
Soybeans	Med_W	$0.990^{***/***}$	$0.988^{***/***}$	$0.988^{\cdot/***}$	$0.980^{\cdot/***}$	$0.986^{/***}$	$0.980^{/***}$	0.966	0.958
	$\bar{W}$	0.985	0.992	0.979	0.975	0.977	0.975	0.979	0.956
	Med_{SR}	$-0.068^{\cdot/***}$	$-0.066^{\cdot/***}$	$-0.067^{\cdot/***}$	$-0.065^{\cdot/***}$	$-0.063^{/***}$	$-0.072^{/***}$	-0.074	-0.132
	Med_{mSR}	$-0.066^{\cdot/***}$	$-0.063^{\cdot/***}$	$-0.066^{\cdot/*}$	$-0.065^{\cdot/*}$	$-0.062^{/*}$	$-0.071^{/*}$	-0.071	-0.129

Notes: See note to Table 39.

Table 41: Out-of-sample terminal wealth and Sharpe ratios – $BP_{90\%}$ over active periods ($q_{75\%}$, first two moments only)

		LIRA				CRRA			
		LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$	*EW*	*MV*
Brent	Med_W	$1.025^{***/***}$	$1.024^{***/***}$	$1.002^{***/***}$	0.989	$1.010^{***/***}$	$1.002^{***/***}$	0.974	0.983
	$\bar{W}$	1.028	1.028	1.002	0.978	0.995	1.002	0.989	0.978
	Med_{SR}	$0.141^{***/***}$	$0.146^{***/***}$	$0.013^{**/***}$	$-0.026^{**/\cdot}$	$0.069^{**/***}$	$0.014^{**/***}$	-0.021	-0.045
	Med_{mSR}	$0.165^{***/***}$	$0.163^{***/***}$	$0.013^{**/***}$	$-0.026^{**/\cdot}$	$0.070^{***/***}$	$0.014^{**/***}$	-0.021	-0.045
Rice	Med_W	$0.995^{***/***}$	$0.995^{***/***}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	$0.976^{***/**}$	0.944	0.929
	$\bar{W}$	1.000	1.000	0.974	0.974	0.974	0.974	0.944	0.943
	Med_{SR}	$-0.041^{**/**}$	$-0.041^{**/**}$	-0.196	-0.196	-0.196	-0.196	-0.153	-0.173
	Med_{mSR}	-0.066	-0.066	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	$-0.349^{**/**}$	-0.155	-0.167
Soybean oil	Med_W	$1.023^{***/***}$	$1.021^{***/***}$	0.997	$0.978^{***/\cdot}$	$0.987^{**/}$	$0.987^{**/}$	0.994	0.977
	$\bar{W}$	1.025	1.029	0.986	0.973	0.981	0.977	1.016	0.982
	Med_{SR}	$0.146^{***/***}$	$0.141^{***/***}$	$-0.014^{\cdot/*}$	$-0.081^{***/\cdot}$	$-0.034^{**/}$	$-0.031^{***/}$	-0.003	-0.078
	Med_{mSR}	$0.176^{***/***}$	$0.165^{***/***}$	$-0.015^{\cdot/*}$	$-0.081^{***/\cdot}$	$-0.034^{*/}$	$-0.032^{**/}$	-0.003	-0.077
Soybeans	Med_W	$1.012^{***/***}$	$1.012^{***/***}$	$1.000^{*/***}$	$0.980^{\cdot/***}$	$0.999^{*/***}$	$0.980^{/***}$	0.966	0.958
	$\bar{W}$	1.006	1.016	0.985	0.975	0.985	0.975	0.979	0.956
	Med_{SR}	$0.102^{***/***}$	$0.095^{***/***}$	$0.002^{\cdot/***}$	$-0.065^{\cdot/***}$	$0.000^{/***}$	$-0.072^{/***}$	-0.074	-0.132
	Med_{mSR}	$0.106^{***/***}$	$0.098^{***/***}$	$-0.001^{\cdot/***}$	$-0.065^{\cdot/*}$	$-0.002^{/***}$	$-0.071^{/*}$	-0.071	-0.129

Notes: See note to Table 39.

Table 42: Opportunity cost – $BP_{10\%}$ over active periods only ($q_{75\%}$, first two moments only)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	Med_{OC}	-0.007	-0.006	0.010	0.011	0.006	0.009
		σ_{OC}	0.079	0.078	0.070	0.089	–	–
	MV vs	Med_{OC}	-0.005	-0.005	0.003	0.002	-0.003	0.003
		σ_{OC}	0.036	0.036	0.041	0.048	–	–
Rice	EW vs	Med_{OC}	0.000	0.000	0.028	0.028	0.028	0.028
		σ_{OC}	0.037	0.037	0.026	0.025	–	–
	MV vs	Med_{OC}	-0.001	-0.001	0.029	0.030	0.029	0.029
		σ_{OC}	0.018	0.018	0.038	0.038	–	–
Soybean oil	EW vs	Med_{OC}	-0.043	-0.041	-0.030	-0.028	-0.036	-0.027
		σ_{OC}	0.092	0.086	0.078	0.088	–	–
	MV vs	Med_{OC}	-0.024	-0.020	-0.009	-0.011	-0.019	-0.005
		σ_{OC}	0.042	0.043	0.064	0.070	–	–
Soybeans	EW vs	Med_{OC}	-0.027	-0.015	-0.010	-0.006	-0.010	-0.007
		σ_{OC}	0.066	0.055	0.060	0.044	–	–
	MV vs	Med_{OC}	-0.005	-0.001	-0.001	0.006	-0.001	0.006
		σ_{OC}	0.028	0.029	0.039	0.050	–	–

Notes: Median opportunity cost (Med_{OC}) and its standard deviation (σ_{OC}) for the $BP_{10\%}$ strategies vs. EW and MV benchmarks, computed on active out-of-sample periods only. OC is expressed as a fraction of initial wealth. NA stands for zero active periods. No σ_{OC} is available for CRRA (denoted –).

Table 43: Opportunity cost – $BP_{50\%}$ over active periods only ($q_{75\%}$, first two moments only)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	Med_{OC}	0.020	0.019	0.015	0.011	0.024	0.015
		σ_{OC}	0.079	0.079	0.070	0.089	–	–
	MV vs	Med_{OC}	0.021	0.020	0.022	0.002	0.009	0.021
		σ_{OC}	0.040	0.039	0.046	0.048	–	–
Rice	EW vs	Med_{OC}	0.039	0.039	0.028	0.028	0.028	0.028
		σ_{OC}	0.027	0.027	0.026	0.025	–	–
	MV vs	Med_{OC}	0.042	0.042	0.029	0.030	0.029	0.029
		σ_{OC}	0.037	0.037	0.038	0.038	–	–
Soybean oil	EW vs	Med_{OC}	-0.000	-0.002	-0.024	-0.028	-0.024	-0.027
		σ_{OC}	0.092	0.090	0.082	0.088	–	–
	MV vs	Med_{OC}	0.017	0.021	-0.002	-0.011	-0.008	-0.005
		σ_{OC}	0.053	0.051	0.067	0.070	–	–
Soybeans	EW vs	Med_{OC}	0.009	0.020	0.004	-0.006	0.001	-0.007
		σ_{OC}	0.051	0.052	0.050	0.044	–	–
	MV vs	Med_{OC}	0.020	0.027	0.010	0.006	0.009	0.006
		σ_{OC}	0.042	0.039	0.042	0.050	–	–

Notes: See note to table 42.

Table 44: Opportunity cost – $BP_{90\%}$ over active periods only ($q_{75\%}$, first two moments only)

			LIRA				CRRA	
			LIRAGauss	LIRADExp	LIRAUnif	LIRAPow3	$\gamma = 5$	$\gamma = 10$
Brent	EW vs	Med_{OC}	0.045	0.046	0.019	0.011	0.035	0.018
		σ_{OC}	0.083	0.082	0.072	0.089	–	–
	MV vs	Med_{OC}	0.049	0.050	0.032	0.002	0.018	0.031
		σ_{OC}	0.037	0.037	0.051	0.048	–	–
Rice	EW vs	Med_{OC}	0.062	0.062	0.028	0.028	0.028	0.028
		σ_{OC}	0.013	0.013	0.026	0.025	–	–
	MV vs	Med_{OC}	0.061	0.061	0.029	0.030	0.029	0.029
		σ_{OC}	0.035	0.035	0.038	0.038	–	–
Soybean oil	EW vs	Med_{OC}	0.036	0.039	-0.024	-0.028	-0.024	-0.027
		σ_{OC}	0.099	0.091	0.086	0.088	–	–
	MV vs	Med_{OC}	0.041	0.046	-0.000	-0.011	-0.002	-0.005
		σ_{OC}	0.048	0.050	0.072	0.070	–	–
Soybeans	EW vs	Med_{OC}	0.036	0.047	0.011	-0.006	0.011	-0.007
		σ_{OC}	0.050	0.053	0.051	0.044	–	–
	MV vs	Med_{OC}	0.040	0.052	0.016	0.006	0.017	0.006
		σ_{OC}	0.043	0.042	0.044	0.050	–	–

Notes: See note to table 42.